Изводи елементарних функција

$$\mathrm{D}f(x)=\frac{\mathrm{d}}{\mathrm{d}x}f(x)=f'(x)=\frac{\mathrm{d}f(x)}{\mathrm{d}x}$$

$C=\mathrm{const}$

$f(x)$	$\frac{\mathrm{d}}{\mathrm{d}x}f(x)$	$f(x)$	$\frac{\mathrm{d}}{\mathrm{d}x}f(x)$	$f(x)$	$\frac{\mathrm{d}}{\mathrm{d}x}f(x)$
C	0	$\sin x$	$\cos x$	$\mathrm{arcsec}\, x$	$\frac{1}{x\sqrt{x^2-1}}$
x	1	$\cos x$	$-\sin x$	$\mathrm{arccosec}\, x$	$\frac{-1}{x\sqrt{x^2-1}}$
x^n	$n\,x^{n-1}$	$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh x$	$\cosh x$
$\frac{1}{x}$	$\frac{-1}{x^2}$	$\cot x$	$\frac{-1}{\sin^2 x}$	$\cosh x$	$\sinh x$
$\frac{1}{x^n}$	$\frac{-n}{x^{n+1}}$	$\sec x$	$\frac{\sin x}{\cos^2 x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	$\mathrm{cosec}\, x$	$\frac{-\cos x}{\sin^2 x}$	$\coth x$	$\frac{-1}{\sinh^2 x}$
e^x	e^x	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	$\mathrm{Areasinh}\, x$	$\frac{1}{\sqrt{1+x^2}}$
a^x	$a^x \ln a$	$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$	$\mathrm{Areacosh}\, x$	$\frac{1}{\sqrt{x^2-1}}$
$\ln x$	$\frac{1}{x}$	$\arctan x$	$\frac{1}{1+x^2}$	$\mathrm{Areatanh}\, x$	$\frac{1}{1-x^2}$
$\log_a x$	$\frac{1}{x\ln a}$	$\mathrm{arccot}\, x$	$\frac{-1}{1+x^2}$	$\mathrm{Areacoth}\, x$	$\frac{1}{1-x^2}$

$a,b=\mathrm{const}$

$$(a\,u+b\,v)'=a\,u'+b\,v'=a\,\mathrm{D}u+b\,\mathrm{D}v$$

$$(u\,v)'=u'v+uv'=(\mathrm{D}u)\,v+u\,(\mathrm{D}v)$$

$$\left(\frac{u}{v}\right)'=\frac{u'v-u\,v'}{v^2}=\frac{(\mathrm{D}u)\,v-u\,(\mathrm{D}v)}{v^2}$$

Интеграли елементарних функција

$f(x)$	$\int f(x)\,\mathrm{d}x$	$f(x)$	$\int f(x)\,\mathrm{d}x$
x^a	$\dfrac{x^{a+1}}{a+1}$, $(a \neq -1)$	$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\arctan\dfrac{x}{a}$, $(a \neq 0)$
$\dfrac{1}{x}$	$\ln\lvert x\rvert$	$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert$, $(a \neq 0)$
e^x	e^x	$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$, $(a \neq 0)$
a^x	$\dfrac{a^x}{\ln a}$	$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\dfrac{x}{a}$, $(\lvert x\rvert < a)$
$\sin x$	$-\cos x$	$\dfrac{1}{\sqrt{a^2+x^2}}$	$\ln\left(x+\sqrt{a^2+x^2}\right)$
$\cos x$	$\sin x$	$\dfrac{1}{\sqrt{x^2-a^2}}$	$\ln\left\lvert x+\sqrt{x^2-a^2}\right\rvert$, $(\lvert x\rvert > a)$
$\tan x$	$-\ln\lvert\cos x\rvert$		
$\cot x$	$\ln\lvert\sin x\rvert$	$x\sin ax$	$\dfrac{1}{a^2}(\sin ax - ax\cos ax)$
$\sinh x$	$\cosh x$	$x\cos ax$	$\dfrac{1}{a^2}(\cos ax + ax\sin ax)$
$\cosh x$	$\sinh x$	$x\,\mathrm{e}^{ax}$	$\dfrac{1}{a^2}(ax-1)\,\mathrm{e}^{ax}$
$\tanh x$	$\ln\lvert\cosh x\rvert$	$e^{ax}\sin bx$	$\dfrac{1}{a^2+b^2}(a\sin bx - b\cos bx)\,\mathrm{e}^{ax}$
$\coth x$	$\ln\lvert\sinh x\rvert$	$e^{ax}\cos bx$	$\dfrac{1}{a^2+b^2}(a\cos bx + b\sin bx)\,\mathrm{e}^{ax}$

$a, b = \mathrm{const}$

$\int (a\,f(x) + b\,g(x))\,\mathrm{d}x = a\int f(x)\,\mathrm{d}x + b\int g(x)\,\mathrm{d}x$, (линеарност)

$\int u(x)v'(x)\,\mathrm{d}x = u(x)v(x) - \int u'(x)v(x)\,\mathrm{d}x$, (парцијална интеграција)

$\int f(g(x))\,g'(x)\,\mathrm{d}x = \int f(z)\,\mathrm{d}z$, (правило смене)

$\int_a^b f(x)\,\mathrm{d}x = F(x)\Big|_a^b = F(b) - F(a)$, (одређени интеграл)

$\int_a^b u(x)v'(x)\,\mathrm{d}x = u(x)v(x)\Big|_a^b - \int_a^b u'(x)v(x)\,\mathrm{d}x$, (парцијална интеграција)

$$\int_a^b f(g(x))\,g'(x)\,\mathrm{d}x = \int_{g(a)}^{g(b)} f(z)\,\mathrm{dz}\,,\ z = g(x)\,,\ \text{(правило смене)}$$

$$\int_\alpha^\beta f(z)\,\mathrm{dz} = \int_{h(\alpha)}^{h(\beta)} f(g(x))\,g'(x)\,\mathrm{d}x\,,\ x = h(z)\,,\ \text{(правило смене)}$$

Тригонометријски идентитети

$\sin(-x) = -\sin x$, $\cos(-x) = \cos x$, $\tan(-x) = -\tan x$, $\cot(-x) = -\cot x$

$\sin x = \cos(x - \frac{\pi}{2})$, $\cos x = -\sin(x - \frac{\pi}{2})$, $\cot x = -\tan(x - \frac{\pi}{2})$

$\sin(x + \frac{\pi}{2}) = \cos x$, $\sin(x + \pi) = \sin(x - \pi) = -\sin x$

$\cos(x + \frac{\pi}{2}) = -\sin x$, $\cos(x + \pi) = \cos(x - \pi) = -\cos x$

$\tan(x + \frac{\pi}{2}) = -\cot x$, $\tan(x + \pi) = \tan(x - \pi) = \tan x$

$\cot(x + \frac{\pi}{2}) = -\tan x$, $\cot(x + \pi) = \cot(x - \pi) = \cot x$

$\sin^2 x + \cos^2 x = 1$, $\cot x = \dfrac{\cos x}{\sin x} = \dfrac{1}{\tan x}$, $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{1}{\cot x}$

$\operatorname{cosec} x = \dfrac{1}{\sin x}$, $\sec x = \dfrac{1}{\cos x}$, $\sec^2 x - \tan^2 x = 1$

$\sin(x + y) = \sin x \cos y + \cos x \sin y$, $\sin(x - y) = \sin x \cos y - \cos x \sin y$

$\cos(x + y) = \cos x \cos y - \sin x \sin y$, $\cos(x - y) = \cos x \cos y + \sin x \sin y$

$\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$, $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

$\cot(x + y) = \dfrac{\cot x \cot y - 1}{\cot y + \cot x}$, $\cot(x - y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$

$$\sin 2x = 2\sin x \cos x = \frac{2\tan x}{1 + \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1 = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$\sin x + \sin y = 2\sin\dfrac{x + y}{2}\cos\dfrac{x - y}{2}$, $\sin x - \sin y = 2\cos\dfrac{x + y}{2}\sin\dfrac{x - y}{2}$

$\cos x + \cos y = 2\cos\dfrac{x + y}{2}\cos\dfrac{x - y}{2}$, $\cos x - \cos y = 2\sin\dfrac{x + y}{2}\sin\dfrac{y - x}{2}$

$\cos x + \sin x = \sqrt{2}\cos(x - \frac{\pi}{4})$, $\cos x - \sin x = \sqrt{2}\cos(x + \frac{\pi}{4})$

$2\sin x \sin y = \cos(x - y) - \cos(x + y)$, $2\cos x \cos y = \cos(x - y) + \cos(x + y)$

$2\sin x \cos y = \sin(x - y) + \sin(x + y)$, $2\cos x \sin y = \sin(x + y) - \sin(x - y)$

$2\sin^2 x = 1 - \cos 2x$, $2\cos^2 x = 1 + \cos 2x$

$\sin 0 = 0$, $\sin\frac{\pi}{6} = \frac{1}{2}$, $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{2} = 1$

$\cos 0 = 1$, $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, $\cos\frac{\pi}{3} = \frac{1}{2}$, $\cos\frac{\pi}{2} = 0$

$\tan 0 = 0$, $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, $\tan\frac{\pi}{4} = 1$, $\tan\frac{\pi}{3} = \sqrt{3}$, $\lim\limits_{x\to\frac{\pi}{2}\pm} \tan x = \mp\infty$

$$\sum_{k=0}^{n-1}\sin\left(x + k\frac{2\pi}{n}\right) = 0, \ (n \in \mathbf{N},\ n > 1), \qquad \sum_{k=0}^{n-1}\sin\left(x - k\frac{2\pi}{n}\right) = 0$$

Комплексни бројеви

$\underline{Z} = R + \mathrm{j}X = Z\,\mathrm{e}^{\mathrm{j}\varphi}$, $(R, X, Z, \varphi \in \mathbf{R})$, $\underline{Z} \in \mathbf{C}$

$Z = \mathrm{mod}\,\underline{Z} = |\underline{Z}| = \mathrm{abs}\,\underline{Z} = \sqrt{R^2 + X^2}$, $\quad \varphi = \angle\underline{Z} = \arg\underline{Z} \in (-\pi, \pi]$

Квадрант	$\varphi \in (-\pi, \pi]$	R, X	$\varphi \in (-\pi, \pi]$
I, IV, $R > 0$	$\arctan\frac{X}{R}$	$R > 0$, $X = 0$	0
II, $R < 0$, $X > 0$	$\pi + \arctan\frac{X}{R}$	$R < 0$, $X = 0$	π
III, $R < 0$, $X < 0$	$-\pi + \arctan\frac{X}{R}$	$R = 0$, $X > 0$	$\frac{\pi}{2}$
		$R = 0$, $X < 0$	$\frac{-\pi}{2}$

$\mathrm{j} = \sqrt{-1}$

$1 = \mathrm{e}^{\mathrm{j}0}$, $-1 = \mathrm{e}^{\mathrm{j}\pi}$, $\mathrm{j} = \mathrm{e}^{\mathrm{j}\pi/2}$, $-\mathrm{j} = \mathrm{e}^{-\mathrm{j}\pi/2}$

$\mathrm{e}^{\mathrm{j}\varphi} = \mathrm{e}^{\mathrm{j}(\varphi + 2k\pi)}$, $\mathrm{e}^{\mathrm{j}\varphi} = \cos\varphi + \mathrm{j}\sin\varphi = \exp(\mathrm{j}\varphi)$

$\underline{a} = \exp(\mathrm{j}2\pi/3)$, $\underline{a} = \frac{-1}{2} + \mathrm{j}\frac{\sqrt{3}}{2}$, $\underline{a}^3 = 1$, $\underline{a}^{-3} = 1$

$1 + \underline{a} + \underline{a}^2 = 0$, $1 + \underline{a}^{-1} + \underline{a}^{-2} = 0$, $\underline{a}^{-1} = \underline{a}^2$, $\underline{a}^{-2} = \underline{a}$

$1 - \underline{a} = \sqrt{3}\exp(-\mathrm{j}\pi/6)$, $\underline{a}^{-1} = \underline{a}^*$, $\frac{1}{\underline{a}} = \underline{a}^*$

$1 - \underline{a}^{-1} = \sqrt{3}\exp(\mathrm{j}\pi/6)$, $\underline{a}^{3n} = 1$, $(n \in \mathbf{Z})$

$\pi = 3{,}141592654\ldots$, $\mathrm{e} = 2{,}718281828\ldots$

$c_0 = 299792458\,\frac{\mathrm{m}}{\mathrm{s}}$, $\mu_0 = 4\pi\,10^{-7}\,\frac{\mathrm{H}}{\mathrm{m}}$, $\varepsilon_0 = \frac{1}{c_0^2\mu_0} = 8{,}854187818\cdot 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}}$

Хиперболичке функције

$$\cosh(\underline{z}) = \frac{e^{\underline{z}} + e^{-\underline{z}}}{2},\ \sinh(\underline{z}) = \frac{e^{\underline{z}} - e^{-\underline{z}}}{2},\ \tanh(\underline{z}) = \frac{e^{\underline{z}} - e^{-\underline{z}}}{e^{\underline{z}} + e^{-\underline{z}}},\ \coth(\underline{z}) = \frac{1}{\tanh(\underline{z})}$$

$$\operatorname{sech}(\underline{z}) = \frac{1}{\cosh(\underline{z})},\ \operatorname{cosech}(\underline{z}) = \frac{1}{\sinh(\underline{z})}$$

$$\sinh(a + jb) = \sinh(a)\cos(b) + j\cosh(a)\sin(b),\ (a, b \in \mathbf{R}),\ j = \sqrt{-1}$$

$$\cosh(a + jb) = \cosh(a)\cos(b) + j\sinh(a)\sin(b),\ (a, b \in \mathbf{R}),\ j = \sqrt{-1}$$

$$\cosh(\underline{z})^2 - \sinh(\underline{z})^2 = 1$$

$$\sinh(-\underline{z}) = -\sinh(\underline{z}),\ \tanh(-\underline{z}) = -\tanh(\underline{z}),\ \coth(-\underline{z}) = -\coth(\underline{z}),$$
$$\cosh(-\underline{z}) = \cosh(\underline{z})$$

$$\sinh(\underline{x} \pm \underline{y}) = \sinh(\underline{x})\cosh(\underline{y}) \pm \cosh(\underline{x})\sinh(\underline{y})$$
$$\cosh(\underline{x} \pm \underline{y}) = \cosh(\underline{x})\cosh(\underline{y}) \pm \sinh(\underline{x})\sinh(\underline{y})$$
$$\tanh(\underline{x} \pm \underline{y}) = \frac{\tanh(\underline{x}) \pm \tanh(\underline{y})}{1 \pm \tanh(\underline{x})\tanh(\underline{y})}$$

$$\sinh(2\underline{z}) = 2\sinh(\underline{z})\cosh(\underline{z}),\ \cosh(2\underline{z}) = \sinh(\underline{z})^2 + \cosh(\underline{z})^2$$
$$\tanh(2\underline{z}) = \frac{2\tanh(\underline{z})}{1 + \tanh(\underline{z})^2}$$

$$(\cosh(\underline{z}) \pm \sinh(\underline{z}))^n = \cosh(n\underline{z}) \pm \sinh(n\underline{z})$$

$$\tanh(\frac{\underline{z}}{2}) = \frac{\cosh(\underline{z}) - 1}{\sinh(\underline{z})} = \frac{\sinh(\underline{z})}{\cosh(\underline{z}) + 1}$$

$$\sinh(\underline{x}) \pm \sinh(\underline{y}) = 2\sinh(\frac{\underline{x} \pm \underline{y}}{2})\cosh(\frac{\underline{x} \mp \underline{y}}{2})$$
$$\cosh(\underline{x}) + \cosh(\underline{y}) = 2\cosh(\frac{\underline{x} + \underline{y}}{2})\cosh(\frac{\underline{x} - \underline{y}}{2})$$
$$\cosh(\underline{x}) - \cosh(\underline{y}) = 2\sinh(\frac{\underline{x} + \underline{y}}{2})\sinh(\frac{\underline{x} - \underline{y}}{2})$$
$$\sinh(ja) = j\sin(a),\ \cosh(ja) = \cos(a),\ \tanh(ja) = j\tan(a)$$

Парови Лапласове трансформације

In[1]:=
```
f = {DiracDelta[t], HeavisideTheta[t], t, t^n,
   Exp[-a * t], Sin[b * t], Cos[b * t], Exp[-a * t] * Sin[b * t],
   Exp[-a * t] * Cos[b * t], t * Exp[-a * t], t * Sin[b * t], t * Cos[b * t],
   t * Exp[-a * t] * Sin[b * t], t * Exp[-a * t] * Cos[b * t]};
F = FullSimplify[LaplaceTransform[#, t, s] & /@ f,
   b ∈ Reals && n ∈ Integers && n > 0];
S[e_] := Style[e, 18]; B[e_] := Style[e, 18, Bold];
T[e_] := e // Transpose // TableForm // TraditionalForm;
{S /@ Take[f, 7], B /@ Take[F, 7], S /@ Take[f, -7], B /@ Take[F, -7]} // T
```

Out[1]//TraditionalForm=

$\delta(t)$	$\mathbf{1}$	$e^{-a t}\sin(b t)$	$\frac{b}{(a+s)^2+b^2}$
$\theta(t)$	$\frac{1}{s}$	$e^{-a t}\cos(b t)$	$\frac{a+s}{(a+s)^2+b^2}$
t	$\frac{1}{s^2}$	$t\, e^{-a t}$	$\frac{1}{(a+s)^2}$
t^n	$n!\, s^{-n-1}$	$t\sin(b t)$	$\frac{2 b s}{\left(b^2+s^2\right)^2}$
$e^{-a t}$	$\frac{1}{a+s}$	$t\cos(b t)$	$\frac{s^2-b^2}{\left(b^2+s^2\right)^2}$
$\sin(b t)$	$\frac{b}{b^2+s^2}$	$t\, e^{-a t}\sin(b t)$	$\frac{2 b (a+s)}{\left((a+s)^2+b^2\right)^2}$
$\cos(b t)$	$\frac{s}{b^2+s^2}$	$t\, e^{-a t}\cos(b t)$	$\frac{(a-b+s)(a+b+s)}{\left((a+s)^2+b^2\right)^2}$

In[2]:=
```
F = {s, (c + d * s) / (a + s), (c + d * s) / (s (a + s)),
   (c + d s) / (a + s) ^2, (c + d s) / (s (a + s) ^2),
   (c + d * s) / ((a + s) * (b + s)), (c + d * s) / (s * (a + s) * (b + s)),
   (p + q * s) / ((a + s) (b + s) (c + s)), (c + d * s) / (a^2 + s^2),
   (c + d * s) / (s * (a^2 + s^2)), (c + d * s) / ((a^2 + s^2) * (b^2 + s^2)),
   (c^2 + s^2) / ((a^2 + s^2) * (b^2 + s^2)),
   (c + d * s) / (a^2 + s^2) ^2, (c^2 + s^2) / (a^2 + s^2) ^2,
   (c + d * s) / ((a + s) * (b^2 + s^2)), (c + d * s) / ((a + s) * (b + s) ^2)};
f = Map[InverseLaplaceTransform[#, s, t] &, F // Apart];
{S /@ f, B /@ F} // T
```

Out[2]//TraditionalForm=

$\delta'(t)$	s
$e^{-a t}(c - a d) + d\,\delta(t)$	$\frac{c+d s}{a+s}$
$\frac{e^{-a t}(a d-c)}{a} + \frac{c}{a}$	$\frac{c+d s}{s (a+s)}$
$-e^{-a t}(a d t - c t - d)$	$\frac{c+d s}{(a+s)^2}$
$\frac{e^{-a t}\left(a^2 d t-a c t+c e^{a t}-c\right)}{a^2}$	$\frac{c+d s}{s (a+s)^2}$
$\frac{e^{-a t}(a d-c)}{a-b} + \frac{e^{-b t}(c-b d)}{a-b}$	$\frac{c+d s}{(a+s)(b+s)}$
$\frac{e^{-a t}(c-a d)}{a (a-b)} + \frac{e^{-b t}(b d-c)}{b (a-b)} + \frac{c}{a b}$	$\frac{c+d s}{s (a+s)(b+s)}$
$\frac{e^{-a t}(p-a q)}{(a-b)(a-c)} + \frac{e^{-b t}(b q-p)}{(a-b)(b-c)} - \frac{e^{-c t}(c q-p)}{(a-c)(b-c)}$	$\frac{p+q s}{(a+s)(b+s)(c+s)}$
$\frac{c \sin(a t)+a d \cos(a t)}{a}$	$\frac{c+d s}{a^2+s^2}$
$\frac{c (-\cos(a t))+a d \sin(a t)+c}{a^2}$	$\frac{c+d s}{s \left(a^2+s^2\right)}$
$\frac{-b c \sin(a t)+a c \sin(b t)-a b d \cos(a t)+a b d \cos(b t)}{a b \left(a^2-b^2\right)}$	$\frac{c+d s}{\left(a^2+s^2\right)\left(b^2+s^2\right)}$
$\frac{\left(a^2-c^2\right)\sin(a t)}{a \left(a^2-b^2\right)} + \frac{\left(c^2-b^2\right)\sin(b t)}{b \left(a^2-b^2\right)}$	$\frac{c^2+s^2}{\left(a^2+s^2\right)\left(b^2+s^2\right)}$
$\frac{a^2 d t \sin(a t)+c \sin(a t)-a c t \cos(a t)}{2 a^3}$	$\frac{c+d s}{\left(a^2+s^2\right)^2}$
$\frac{\left(c^2-a^2\right)(\sin(a t)-a t \cos(a t))}{2 a^3} + \frac{\sin(a t)}{a}$	$\frac{c^2+s^2}{\left(a^2+s^2\right)^2}$
$\frac{e^{-a t}(c-a d)}{a^2+b^2} + \frac{a c \sin(b t)+a b d \cos(b t)+b^2 d \sin(b t)-b c \cos(b t)}{b \left(a^2+b^2\right)}$	$\frac{c+d s}{(a+s)\left(b^2+s^2\right)}$
$\frac{e^{-a t}(c-a d)}{(a-b)^2} + \frac{e^{-b t}(a d-c)}{(a-b)^2} + \frac{t e^{-b t}(c-b d)}{a-b}$	$\frac{c+d s}{(a+s)(b+s)^2}$