1. $\displaystyle\int \frac{dx}{\sqrt[4]{(x-1)^3(x+2)^5}}\cdot\frac{\sqrt[4]{x-1}}{\sqrt[4]{x-1}} = \int \frac{1}{(x-1)(x+2)}\sqrt[4]{\frac{x-1}{x+2}}\,dx$

smena : $\dfrac{x-1}{x+2} = t^4$

$x-1 = xt^4 + 2t^4$

$x(1-t^4) = 2t^4+1$

$x = \dfrac{2t^4+1}{1-t^4}$ $/d$

$$dx = \frac{8t^3(1-t^4)-(2t^4+1)\cdot(-4t^3)}{(1-t^4)^2}\,dt$$

$$dx = \frac{8t^3-8t^7+8t^7+4t^3}{(1-t^4)^2}\,dt$$

$$dx = \frac{12t^3\,dt}{(1-t^4)^2}$$

$$\int \frac{1}{\left(\frac{2t^4+1}{1-t^4}-1\right)\left(\frac{2t^4+1}{1-t^4}+2\right)}\cdot t\cdot\frac{12t^3dt}{(1-t^4)^2} = \int \frac{\cancel{(1-t^4)^2}}{(2t^4+1-1+t^4)(2t^4+1+2-2t^4)}\cdot\frac{12t^4dt}{\cancel{(1-t^4)^2}} =$$

$$= \int \frac{\overset{4}{\cancel{12}}\cancel{t^4}\,dt}{\cancel{3}\cancel{t^4}\cdot\cancel{1}\cdot 3} = \frac{4}{3}t + C = \frac{4}{3}\sqrt[4]{\frac{x-1}{x+2}} + C = \frac{4}{3}\frac{x-1}{\sqrt[4]{(x-1)^3(x+2)}}\cdot\frac{x+2}{x+2} + C = \boxed{\frac{4}{3}\frac{(x-1)(x+2)}{\sqrt[4]{(x-1)^3(x+2)^5}} + C}$$

$\cancel{2.\ \int \frac{x^2}{(x\sin x+\cos x)^2}}$

2. $\displaystyle I = \int \frac{x\,e^{\operatorname{arctg} x}}{(1+x^2)^{\frac{3}{2}}}\,dx$

$$d(e^{\operatorname{arctg} x}) = \frac{e^{\operatorname{arctg} x}}{1+x^2}\,dt$$

$$= \int \frac{x}{\sqrt{1+x^2}}\cdot\frac{e^{\operatorname{arctg} x}}{1+x^2}\,dx = \int \underbrace{\frac{x}{\sqrt{1+x^2}}}_{u}\cdot\underbrace{d(e^{\operatorname{arctg} x})}_{dv}$$

1° parcijalna: $u = \dfrac{x}{\sqrt{1+x^2}}$ $\quad dv = d(e^{\operatorname{arctg} x})$

$$du = \frac{\sqrt{1+x^2} - x\cdot\frac{1}{2}\cdot\frac{2x\,dx}{\sqrt{1+x^2}}}{1+x^2} \qquad v = e^{\operatorname{arctg} x}$$

$$du = \frac{\frac{1+x^2-x^2}{\sqrt{1+x^2}}}{1+x^2}\,dx = \frac{dx}{(1+x^2)^{\frac{3}{2}}}$$

$$= \frac{x}{\sqrt{1+x^2}}\cdot e^{\operatorname{arctg} x} - \int \frac{e^{\operatorname{arctg} x}}{(1+x^2)^{\frac{3}{2}}}\,dx$$

2° parcijalna: $\dfrac{1}{\sqrt{1+x^2}} = u$ $/d$ $\quad d(e^{\operatorname{arctg} x}) = dv$ $//$

$$-\frac{1}{2}\,\frac{2x\,dx}{(1+x^2)^{\frac{3}{2}}} = du \qquad e^{\operatorname{arctg} x} = v$$

$$-\frac{x\,dx}{(1+x^2)^{\frac{3}{2}}} = du$$

$$= \frac{x}{\sqrt{1+x^2}}\,e^{\operatorname{arctg} x} - \int \frac{1}{\sqrt{1+x^2}}\cdot d(e^{\operatorname{arctg} x})$$

$$= \frac{x}{\sqrt{1+x^2}}\,e^{\operatorname{arctg} x} - \left(\frac{1}{\sqrt{1+x^2}}\,e^{\operatorname{arctg} x} + \underbrace{\int \frac{x\,e^{\operatorname{arctg} x}}{(1+x^2)^{\frac{3}{2}}}\,dx}_{I}\right)$$

$$\Rightarrow\ 2I = \frac{e^{\operatorname{arctg} x}}{\sqrt{1+x^2}}(x-1)$$

$$\Rightarrow \boxed{I = \frac{1}{2}\,e^{\operatorname{arctg} x}\cdot\frac{x-1}{\sqrt{1+x^2}} + C}$$

3. $\int \frac{\sin x \, dx}{\sin^3 x + \cos^3 x}$

smena: $t = tg x$

$x = arctg t \quad /d$

$dx = \frac{dt}{1+t^2}$

$\sin^2 x + \cos^2 x = 1 \quad / : \cos^2 x$

$tg^2 x + 1 = \frac{1}{\cos^2 x}$

$\cos^2 x = \frac{1}{1+tg^2 x}$

$\cos x = \frac{1}{\sqrt{1+tg^2 x}} = \frac{1}{\sqrt{1+t^2}}$

$\sin^2 x = 1 - \frac{1}{tg^2 x + 1} = \frac{1 + tg^2 x - 1}{1+tg^2 x}$

$\sin^2 x = \frac{tg^2 x}{1+tg^2 x}$

$\sin x = \frac{tg x}{\sqrt{1+tg^2 x}}$

$\sin x = \frac{t}{\sqrt{1+t^2}}$

$$\int \frac{\frac{t}{\sqrt{1+t^2}} \cdot \frac{dt}{1+t^2}}{\frac{t^3}{(1+t^2)\sqrt{1+t^2}} + \frac{1}{(1+t^2)\sqrt{1+t^2}}} = \int \frac{t\,dt}{t^3+1} = \int \frac{t\,dt}{(t+1)(t^2-t+1)}$$

$$\frac{t}{(t+1)(t^2-t+1)} = \frac{A}{t+1} + \frac{Bt+C}{t^2-t+1}$$

$t = At^2 - At + A + Bt^2 + Bt + Ct + C$

$t = t^2(A+B) + t(-A+B+C) + A + C$

$0 = A+B \quad \rightarrow A = -B$

$1 = -A+B+C \quad \rightarrow 1 = C+C+C \qquad C = B = \frac{1}{3}, \; A = -\frac{1}{3}$

$0 = A + C \quad \rightarrow A = -C$

$$-\frac{1}{3}\int \frac{dt}{t+1} + \frac{1}{3}\int \frac{t+1}{t^2-t+1}dt = -\frac{1}{3}\ln|t+1| + \frac{1}{3}\cdot\frac{1}{2}\int \frac{2t+2}{t^2-t+1}dt =$$

$$= -\frac{1}{3}\ln|t+1| + \frac{1}{6}\int \frac{2t-1}{t^2-t+1}dt + \frac{3}{6}\int \frac{dt}{t^2-t+1} \longrightarrow$$

$t^2 - 2\cdot\frac{1}{2}t + \frac{1}{4} - \frac{1}{4} + 1$

$= \left(t-\frac{1}{2}\right)^2 + \frac{3}{4}$

$$= -\frac{1}{3}\ln|t+1| + \frac{1}{6}\ln|t^2-t+1| + \frac{1}{2}\int \frac{dt}{\left(\frac{2t-1}{\sqrt{3}}\right)^2+1} \cdot \frac{4}{3}$$

$= \frac{3}{4}\left(\frac{\cancel{4}}{3}\frac{(2t-1)^2}{\cancel{4}} + 1\right)$

$$= -\frac{1}{3}\ln|t+1| + \frac{1}{3}\ln\sqrt{t^2-t+1} + \frac{\cancel{2}}{3}\int \frac{d\left(\frac{2t-1}{\sqrt{3}}\right)}{\left(\frac{2t-1}{\sqrt{3}}\right)^2+1} \cdot \frac{\sqrt{3}}{\cancel{2}}$$

$= \frac{3}{4}\left(\left(\frac{2t-1}{\sqrt{3}}\right)^2 + 1\right)$

$$= \boxed{\frac{1}{3}\ln\frac{\sqrt{t^2-t+1}}{|t+1|} + \frac{\sqrt{3}}{3}\, arctg\left(\frac{2t-1}{\sqrt{3}}\right) + C}$$

4. $\int x^2 \cdot \sin(1-x)\, dx$

$1 - x = t, \quad x = 1 - t$

$dx = -dt$

$$= \int (1-t)^2 \cdot \sin t \cdot (-dt)$$

$$= -\int (1 - 2t + t^2)\sin t\, dt$$

$$= -\int \sin t\, dt + 2\int t \sin t\, dt - \int t^2 \sin t\, dt$$

$$= \cos t + 2\left(-t\cos t + \int \cos t\, dt\right) - \left(-t^2\cos t + 2\int t\cos t\, dt\right)$$

$$= \cos t - 2t\cos t + 2\sin t + t^2\cos t - 2\int t\cos t\, dt$$

$$= \cos t - 2t\cos t + 2\sin t + t^2\cos t - 2\left(t\sin t - \int \sin t\, dt\right)$$

$$= \cos t - 2t\cos t + 2\sin t + t^2\cos t - 2t\sin t - 2\cos t + C$$

$$= \boxed{t^2\cos t - 2t(\cos t + \sin t) + 2\sin t - \cos t + C}$$

$1^\circ \; t = u \qquad \sin t\, dt = d\upsilon$

$dt = du \qquad -\cos t = \upsilon$

$2^\circ \; t^2 = u \qquad \sin t\, dt = d\upsilon$

$2t\, dt = du \qquad -\cos t = \upsilon$

$3^\circ \; t = u \qquad \cos t\, dt = d\upsilon$

$dt = du \qquad \sin t = \upsilon$

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5. $\displaystyle\int \frac{x\,dx}{\sqrt{1+\sqrt[3]{x^2}}} = \int x\left(1+x^{\frac{2}{3}}\right)^{-\frac{1}{2}}dx$

$\int x^m(a+bx^n)^p dx$

$\frac{m+1}{n} = \cancel{\frac{1+1}{\frac{2}{3}}} = 3 \in Z$

$t^s = a+bx^n$

$\boxed{t = 1+x^{\frac{2}{3}}} \;/d \qquad x^{\frac{2}{3}} = t-1$

$dt = \frac{2}{3}x^{-\frac{1}{3}}dx \qquad x^{\frac{4}{3}} = (t-1)^2$

$= \frac{3}{2}\int x^{\frac{4}{3}}\underbrace{\left(1+x^{\frac{2}{3}}\right)}_{t}{}^{-\frac{1}{2}}\cdot\underbrace{\frac{2}{3}x^{-\frac{1}{3}}dx}_{dt}$

$= \frac{3}{2}\int (t-1)^2\cdot t^{-\frac{1}{2}}dt$

$= \frac{3}{2}\int (t^2-2t+1)\cdot t^{-\frac{1}{2}}dt$

$= \frac{3}{2}\int\left(t^{\frac{3}{2}} - 2t^{\frac{1}{2}} + t^{-\frac{1}{2}}\right)dt = \frac{3}{2}\cdot\frac{1}{1+\frac{3}{2}}\cdot t^{1+\frac{3}{2}} - \cancel{\frac{3}{2}}\cdot\cancel{2}\cdot\frac{1}{1+\frac{1}{2}}t^{1+\frac{1}{2}} + \frac{3}{2}\cdot\frac{1}{1-\frac{1}{2}}t^{1-\frac{1}{2}} + C$

$= \frac{3}{5}t^{\frac{5}{2}} - 2t^{\frac{3}{2}} + 3t^{\frac{1}{2}} + C$

$= \boxed{\frac{3}{5}\left(1+x^{\frac{2}{3}}\right)^{\frac{5}{2}} - 2\left(1+x^{\frac{2}{3}}\right)^{\frac{3}{2}} + 3\left(1+x^{\frac{2}{3}}\right)^{\frac{1}{2}} + C}$

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6. Izvesti rekorentnu formulu i izračunati I_n

$I_n = \int_{-1}^{1}(1-x^2)^n dx \quad , \quad n\in\mathbb{N}$

$(1-x^2)^n = u \;/d \qquad dx = d\vartheta$

$n(1-x^2)^{n-1}\cdot(-2x\,dx) = du \qquad x = \vartheta$

$-2n\,x(1-x^2)^{n-1}dx = du$

$I_n = x(1-x^2)^n\Big|_{-1}^{1} + 2n\int x^2(1-x^2)^{n-1}dx = x(1-x^2)^n - 2n\int(1-x^2-1)(1-x^2)^{n-1}dx$

$= x(1-x^2)^n\Big|_{-1}^{1} - 2n\underbrace{\int(1-x^2)^n dx}_{I_n} + 2n\underbrace{\int(1-x^2)^{n-1}dx}_{I_{n-1}}$

$I_n(1+2n) = \underbrace{x(1-x^2)^n\Big|_{-1}^{1}}_{0} + 2n\,I_{n-1}$

$\boxed{I_n = \frac{2n}{1+2n}I_{n-1}} \qquad I_0 = \int_{-1}^{1}dx = 1-(-1) = 2$

$I_n = \frac{2n}{1+2n}I_{n-1} = \frac{2n}{1+2n}\cdot\frac{2(n-1)}{1+2(n-1)}\cdot I_{n-2} = \ldots = \frac{2n(2n-2)(2n-4)\cdots 2}{(2n+1)(2n-1)(2n-3)\cdots 3}\cdot I_0$

$\boxed{I_n = \frac{2\cdot(2n)!!}{(2n+1)!!}}$

7. $I_n = \int \frac{x^n\,dx}{\sqrt{1+x^2}} = \int \frac{x^{n-1}\cdot x\,dx}{\sqrt{1+x^2}}$

$x^{n-1} = u$ $\qquad \frac{x\,dx}{\sqrt{1+x^2}} = dv \;\; /\int$ $\qquad 1+x^2 = t \;/d$

$(n-1)x^{n-2}dx = du$ $\qquad x\,dx = \frac{dt}{2}$

$\frac{1}{2}\int \frac{dt}{\sqrt{t}} = \frac{1}{2}\cdot 2\sqrt{t} = \sqrt{1+x^2} = v$

$I_n = x^{n-1}\sqrt{1+x^2} - (n-1)\int x^{n-2}\sqrt{1+x^2}\,dx \cdot \frac{\sqrt{1+x^2}}{\sqrt{1+x^2}}$

$= x^{n-1}\sqrt{1+x^2} - (n-1)\int \frac{x^{n-2}(1+x^2)}{\sqrt{1+x^2}}dx$

$= x^{n-1}\sqrt{1+x^2} - (n-1)\underbrace{\int \frac{x^{n-2}}{\sqrt{1+x^2}}dx}_{I_{n-2}} - (n-1)\underbrace{\int \frac{x^n\,dx}{\sqrt{1+x^2}}}_{I_n}$

$I_n(1+n-1) = x^{n-1}\sqrt{1+x^2} + (1-n)I_{n-2}$

$$\boxed{I_n = \frac{x^{n-1}}{n}\sqrt{1+x^2} + \frac{1-n}{n}I_{n-2}}$$

8. $I_n = \int \frac{dx}{\sin^n x} = \int \sin^{-(n-2)}x \cdot \frac{dx}{\sin^2 x}$

$\sin^{2-n}x = u \;/d$ $\qquad \frac{dx}{\sin^2 x} = dv \;\; /\int$

$(2-n)\cdot \sin^{1-n}\underset{\cos x\,dx}{x} = du\,!$ $\qquad -\operatorname{ctg} x = v$

$I_n = -\operatorname{ctg} x\cdot \sin^{2-n}x + (2-n)\int \sin^{1-n}x\cdot\frac{\cos^2 x}{\sin x}dx$

$= -\operatorname{ctg}x\cdot\sin^{2-n}x + (2-n)\int \sin^{-n}x\cdot(1-\sin^2 x)\,dx$

$= -\operatorname{ctg}x\cdot\sin^{2-n}x + (2-n)\underbrace{\int\frac{dx}{\sin^n x}}_{I_n} - (2-n)\underbrace{\int\frac{dx}{\sin^{n-2}x}}_{I_{n-2}}$

$I_n(1-2+n) = -\operatorname{ctg}x\cdot\sin^{2-n}x + (n-2)I_{n-2}$

$$\boxed{I_n = -\frac{1}{n-1}\cdot\operatorname{ctg}x\cdot\sin^{2-n}x + \frac{(n-2)}{n-1}I_{n-2}}$$

9. $I_n = \int\frac{dx}{\cos^n x} = \int \cos^{-(n-2)}x\cdot\frac{dx}{\cos^2 x} \longrightarrow \ldots \boxed{I_n = \frac{1}{n-1}\cdot\operatorname{tg}x\cdot\cos^{2-n}x + \frac{n-2}{n-1}I_{n-2}}$

$\cos^{2-n}x = u$ $\qquad \frac{dx}{\cos^2 x} = dv$

$(2-n)\cos^{1-n}x\cdot(-\sin x)\,dx = du$ $\qquad \operatorname{tg}x = v$

10. $I_n = \int x^n \cos x \, dx$

$$\left[\begin{array}{ll} 1^\circ \quad x^n = u & \cos x\, dx = dv \\ n x^{n-1} dx = du & \sin x = v \end{array}\right] \quad \left[\begin{array}{ll} 2^\circ \quad x^{n-1} = u & \sin x\, dx = dv \\ (n-1) x^{n-2} dx = du & -\cos x = v \end{array}\right]$$

$$I_n = x^n \sin x - n \int x^{n-1} \cdot \cancel{\cos} \sin x \, dx$$

$$= x^n \sin x - n \left(-x^{n-1} \cdot \cos x + (n-1) \int \underbrace{x^{n-2} \cdot \cos x\, dx}_{I_{n-2}}\right)$$

$$\boxed{I_n = x^n \sin x + n x^{n-1} \cdot \cos x - n(n-1) I_{n-2}}$$

11. $I_n = \int x^n \sin x \, dx \longrightarrow \ldots \boxed{I_n = -x^n \cos x + n x^{n-1} \sin x - n(n-1) I_{n-2}}$

$x^n = u \quad , \quad \sin x\, dx = dv$

12. $I_n = \int x^n e^{\sqrt{x}} dx \qquad d(e^{\sqrt{x}}) = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} dx$

$$I_n = 2 \int x^{n+\frac{1}{2}} \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} dx = 2 \int x^{n+\frac{1}{2}} \cdot d(e^{\sqrt{x}})$$

$$\left[\begin{array}{ll} x^{n+\frac{1}{2}} = u & d(e^{\sqrt{x}}) = dv \quad \int \\ (n+\frac{1}{2}) x^{n-\frac{1}{2}} dx = du & e^{\sqrt{x}} = v \end{array}\right]$$

$$I_n = 2 x^{n+\frac{1}{2}} e^{\sqrt{x}} - (2n+1) \int x^{n-\frac{1}{2}} e^{\sqrt{x}} dx \longrightarrow 2 \int \frac{x^n e^{\sqrt{x}}}{2\sqrt{x}} dx$$

$$\left[\begin{array}{ll} x^n = u & d(e^{\sqrt{x}}) = dv \\ n x^{n-1} dx = du & e^{\sqrt{x}} = v \end{array}\right]$$

$$I_n = 2 x^{n+\frac{1}{2}} e^{\sqrt{x}} - 2(2n+1) \left[x^n e^{\sqrt{x}} - n \int \underbrace{x^{n-1} \cdot e^{\sqrt{x}} dx}_{I_{n-1}} \right]$$

$$\boxed{I_n = 2 x^{n+\frac{1}{2}} e^{\sqrt{x}} - 2(2n+1) x^n e^{\sqrt{x}} + 2n(2n+1) I_{n-1}}$$

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13. $\displaystyle\int_2^{+\infty} \frac{\operatorname{arctg} x}{(1-x)^2}\,dx$ $\left[\begin{array}{ll} \operatorname{arctg} x = u \quad /d & \dfrac{dx}{(1-x)^2} = dv \\ \dfrac{dx}{1+x^2} = du & \dfrac{1}{1-x} = v \end{array}\right]$

$$= \frac{1}{1-x}\cdot \operatorname{arctg} x \Big|_2^{+\infty} - \int_2^{+\infty} \frac{dx}{(1-x)(1+x^2)}$$

$$\frac{1}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$$

$$= \lim_{b\to\infty} \cancel{\frac{\operatorname{arctg} b}{1-b}}^{\to 0} - \frac{1}{1-2}\cdot \operatorname{arctg} 2$$

$$1 = Ax^2 + A \cancel{*} Bx^2 + Bx - Cx + C$$

$$- \frac{1}{2}\int_2^{+\infty} \frac{dx}{1-x} - \frac{1}{2}\int_2^{+\infty} \frac{x+1}{x^2+1}\,dx$$

$$\left.\begin{array}{l} 0 = A - B \\ 0 = B - C \\ 1 = A + C \end{array}\right\} A = B = C = \frac{1}{2}$$

$$= \operatorname{arctg} 2 + \frac{1}{2}\ln|1-x| - \frac{1}{4}\int_2^{+\infty} \frac{2x\,dx}{x^2+1} - \frac{1}{2}\int_2^{+\infty} \frac{dx}{x^2+1}$$

$$= \operatorname{arctg} 2 + \left(\frac{1}{2}\ln|1-x| - \frac{1}{4}\ln|x^2+1| - \frac{1}{2}\operatorname{arctg} x\right)\Big|_2^{+\infty}$$

$$= \operatorname{arctg} 2 + \left(\frac{1}{2}\ln\frac{|1-x|}{\sqrt{x^2+1}} - \frac{1}{2}\operatorname{arctg} x\right)\Big|_2^{+\infty}$$

$$= \operatorname{arctg} 2 + \frac{1}{2}\ln \underbrace{\lim_{b\to+\infty} \frac{|1-b|}{\sqrt{b^2+1}}}_{\to 1} - \frac{1}{2}\ln\frac{|1-2|}{\sqrt{2^2+1}} - \frac{1}{2}\cdot\frac{\pi}{2} + \frac{1}{2}\operatorname{arctg} 2$$

$$= \frac{3}{2}\operatorname{arctg} 2 - \frac{\pi}{4} + \cancel{\frac{1}{2}\ln 1}^{\,0} - \frac{1}{2}\ln\frac{1}{\sqrt{5}}$$

$$= \boxed{\frac{3}{2}\operatorname{arctg} 2 - \frac{\pi}{4} - \frac{1}{2}\ln\frac{1}{\sqrt{5}}}$$

14. Izračunati površinu figure ograničene grafikom fje $f(x) = x\sqrt{2x-x^2}$ i x-osom

$2x - x^2 \geq 0$

$x(2-x) \geq 0$

$\Rightarrow x \in [0,2]$

nule : $x=0$, $x=2$

$$\int_0^2 x\sqrt{2x-x^2}\,dx = \int_0^2 x\sqrt{\cancel{*}(x^2-2x+1-1)}\,dx = \int_0^2 x\sqrt{1-(x-1)^2}\,dx$$

$\left[\begin{array}{l} x-1 = \sin t \\ dx = \cos t\,dt \end{array}\right.$

$0 \to -\frac{\pi}{2}$

$2 \to \frac{\pi}{2}$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin t + 1)\sqrt{1-\sin^2 t}\cdot\cos t\,dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin t + 1)\cos^2 t\,dt$$

podintegralna neparna fja sa simetričnim granicama $= 0$

$$= \underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t\cdot\sin t\,dt}_{0} + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t\,dt$$

$$= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1+\cos 2t)\,dt = \frac{1}{2}\cdot\pi + \frac{1}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos 2t\,d(2t)$$

$$= \frac{\pi}{2} + \underbrace{\frac{1}{4}\sin 2t\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}_{0} = \boxed{\frac{\pi}{2}}$$