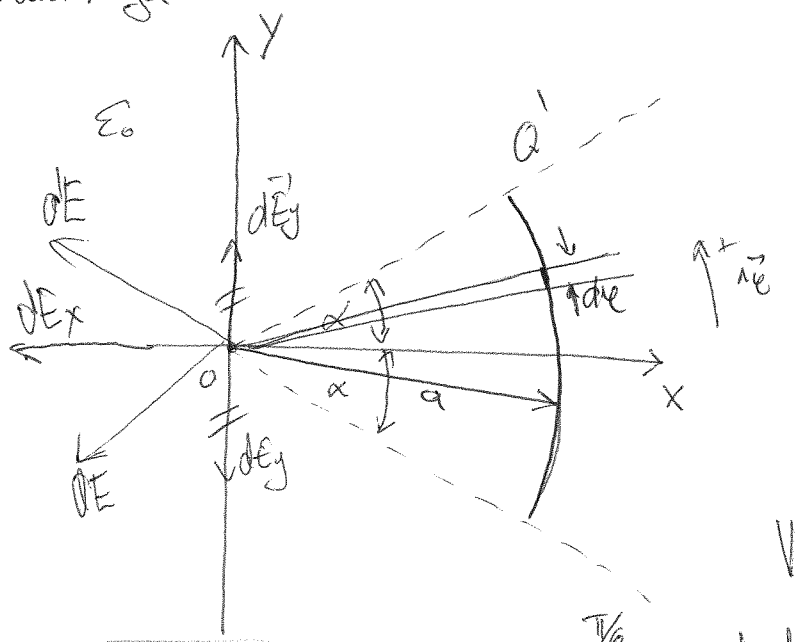
Александар Дацић

24. новембар . 2012

1. задатак



$\alpha = \pi/6$, a , Q'

dE_y компонента неће постојати јер ће се кратити због симетрије.

а) $V_0 = ?$ б) $E_0 = ?$

$dQ = Q' a \, d\varphi$

$V = \frac{Q}{4\pi\varepsilon_0 r} \Rightarrow dV = \frac{dQ}{4\pi\varepsilon_0 a}$

$$V_0 = \int_{-\pi/6}^{\pi/6} \frac{Q' a \, d\varphi}{4\pi\varepsilon_0 a} = \frac{Q'}{4\pi\varepsilon_0 a} \int_{-\pi/6}^{\pi/6} d\varphi \qquad \pi/6 - (-\pi/6) = \pi/3$$

$$\boxed{V_0 = \frac{Q'}{12\varepsilon_0}}$$

Теорија: $E = \frac{Q}{4\pi\varepsilon_0 r^2}$

$V = \frac{Q}{4\pi\varepsilon_0 r}$

$dE_0 = \frac{dQ}{4\pi\varepsilon_0 a^2}$

$dQ = Q' dl = Q' a \, d\varphi$

$dE_{0x} = dE_0 \cdot \cos\varphi$

$dE_{0x} = \frac{Q' \alpha}{4\pi\varepsilon_0 a^2} \cos\varphi \, d\varphi$

$$E_{0x} = \int_{-\pi/6}^{\pi/6} \underbrace{\frac{Q'}{4\pi\varepsilon_0 a}}_{\text{const.}} \cos\varphi \, d\varphi = \frac{Q'}{4\pi\varepsilon_0 a} \int_{-\pi/6}^{\pi/6} \cos\varphi \, d\varphi \qquad 1$$

$E_{0x} = \frac{Q'}{4\pi\varepsilon_0 a}$ = интензитет електричног поља

$$\boxed{\vec{E}_{0x} = -\frac{Q'}{4\pi\varepsilon_0 a} \vec{n}_x}$$ = вектор електричног поља

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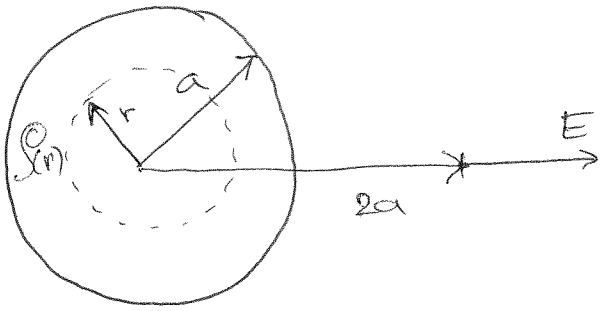
2. питалица



$$dQ = \rho dV = \rho 4r^2\pi dr$$

$$dV = 4r^2\pi dr$$

$$Q = \int_a^a dQ = \int_0^a k r 4r^2\pi dr$$

$$= 4k\pi \frac{r^4}{4}\Big|_0^a = k\pi a^4$$

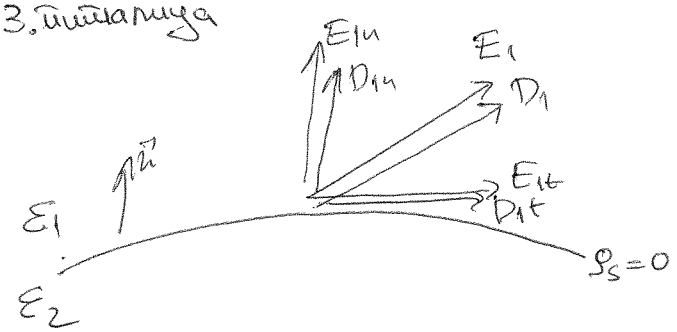
$$Q = k\pi a^4$$

$$\oint_S \vec{E}d\vec{S} = \frac{Q_{us}}{\varepsilon_0}$$

$$E_0 4\cdot(2a)^2\pi = \frac{k\pi a^4}{\varepsilon_0}$$

$$E_0 4\cdot 4\cdot a^2 = \frac{k a^4}{\varepsilon_0} \Rightarrow \boxed{k = \frac{16\varepsilon_0 E_0}{a^2}}$$

3. питалица



Граничини услови

① $E_{1t} = E_{2t}$

② $D_{1n} - D_{2n} = \rho_s = 0$

$D_{1n} = D_{2n}$

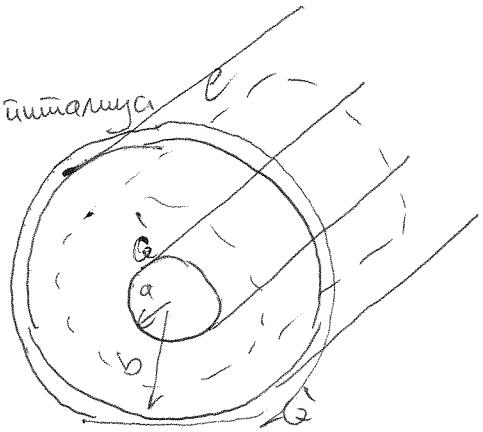
$$D_{1n} = \varepsilon_1 E_{1n} \Rightarrow E_{1n} = \frac{D_{1n}}{\varepsilon_1}$$

$$D_{2n} = \varepsilon_2 E_{2n} \Rightarrow E_{2n} = \frac{D_{2n}}{\varepsilon_2}$$

$$\frac{tg\alpha_1}{tg\alpha_2} = \frac{\frac{E_{1t}}{E_{1n}}}{\frac{E_{2t}}{E_{2n}}} = \frac{E_{2n}}{E_{1n}} = \frac{\frac{D_{2n}}{\varepsilon_2}}{\frac{D_{1n}}{\varepsilon_1}}$$

$$\boxed{\frac{tg\alpha_1}{tg\alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}}$$

4. питалица



$$\oint \vec{D}d\vec{S} = Q_{us}$$

$$D(r)\cdot 2\pi r\cdot \ell = Q'\cdot \ell$$

$$D(r) = \frac{Q'}{2\pi r}$$

$$D(r) = \varepsilon(r) E(r)$$

$$E(r) = \frac{D(r)}{\varepsilon(r)} = \frac{Q'}{2\pi r 2\varepsilon_0 b}$$

$$E(r) = \frac{Q'}{4\pi\varepsilon_0 b} *$$

$$U = \int_a^b E(r)dr = \int_a^b \frac{Q'}{4\pi\varepsilon_0 b} = \frac{Q'}{4\pi\varepsilon_0 b}(b-a)$$

a, b

$E_{kr} = 20\ MV/m$

$\varepsilon(r) = 2\varepsilon_0 \frac{b}{r}$

$D_{1n} = D_{2n} = D(r)$

$U = \int_a^b \vec{E}d\vec{r}$

$$E_{max} = E_{kr} = \frac{Q'_{max}}{4\pi\varepsilon_0 b} \Rightarrow Q'_{max} = 4\pi\varepsilon_0 b E_{kr}$$

$$U_{kr} = E_{kr}(b-a) = 20\ MV/m \cdot (3mm - 1mm) = 20\ \frac{kV}{mm}\cdot 2mm = 40\ kV/m$$

$$\boxed{|U_{kr}| = 40\ kV/m}$$

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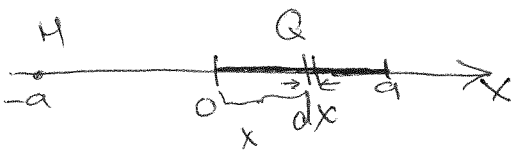
12. Новембар 2011

1. питања



$V = \frac{Q}{4\pi\varepsilon_0 r}$ теорија

$dV = \frac{dQ}{4\pi\varepsilon_0(a+x)}$

$Q' = \frac{Q}{a}$ $\quad dQ = Q'dx$

$dQ = \frac{Q}{a}dx$

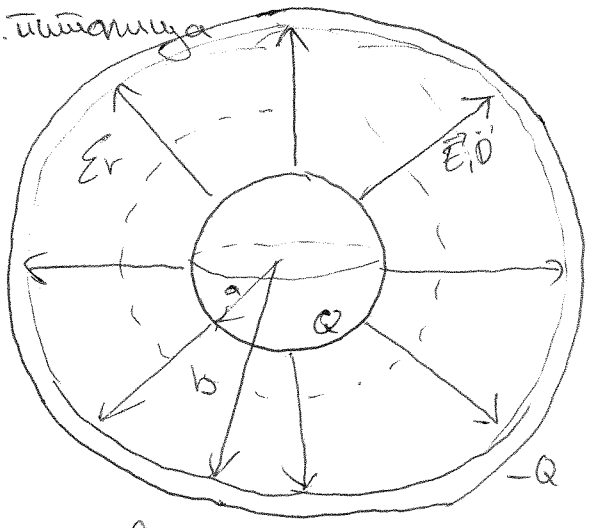
$dV = \frac{Q\,dx}{4\pi\varepsilon_0 a(a+x)} \Rightarrow V = \int_0^a dV = \int_0^a \frac{Q}{4\pi\varepsilon_0 a}\frac{dx}{a+x}$ (const.)

$V = \frac{Q}{4\pi\varepsilon_0 a}\ln(a+x)\Big|_0^a = \frac{Q}{4\pi\varepsilon_0 a}(\ln 2a - \ln a)$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 a}\ln 2}$$

2. питања $\Rightarrow$ теорија $D_{1n} = D_{2n}$ (1636 - 137 страница)

3. питања



$b = 100\,mm \quad \varepsilon_r = 3 \quad E_{kr} = 20\,MV/m$

$U = \int_a^b E(r)dr$

$\oint_S \vec{D}d\vec{s} = Q_{us}$

$D(r) = \varepsilon_0\varepsilon_r E(r) \Rightarrow$

$D(r)\,4r^2\pi = Q \Rightarrow E(r) = \frac{D(r)}{\varepsilon_0\varepsilon_r}$

$D(r) = \frac{Q}{4r^2\pi} \quad E(r) = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}$

$U = \int_a^b \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}dr = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left(-\frac{1}{r}\Big|_a^b\right)$ (const.)

$U = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left(-\frac{1}{b}+\frac{1}{a}\right) \Rightarrow U = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\cdot\frac{b-a}{ab}$

$E_{kr} = \frac{Q_{max}}{4\pi\varepsilon_0\varepsilon_r a^2}$

$Q_{max} = E_{kr}\,4\pi\varepsilon_0\varepsilon_r a^2$

$U_{kr} = \frac{aE_{kr}}{b}(b-a)$

$\frac{dU_{kr}}{da} = 0 \quad \frac{(ab-a^2)'b - (ab-a^2)b'}{b^2} = 0$

$b - 2a = 0 \Rightarrow \boxed{a = \frac{b}{2}}$

$U_{kr\,max} = \frac{\frac{b}{2}E_{kr}(b-\frac{b}{2})}{b}$

$$\boxed{U_{kr\,max} = \frac{E_{kr}b}{4}}$$

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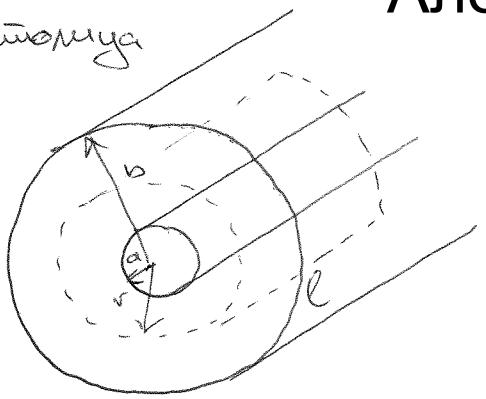
4. питомца



$\varepsilon_r(a)=5 \quad w_e=\text{const.}$

$w_e=\frac{1}{2}DE=\frac{1}{2}\varepsilon_0\varepsilon_r(r)E^2$

$\oint_S \vec{D}d\vec{S}=Q_{us}$

$D(r)\cdot 2\pi r\cdot \ell = Q'\cdot \ell$

$D(r)=\frac{Q'}{2\pi r} \Rightarrow E(r)=\frac{Q'}{2\pi\varepsilon_0\varepsilon_r(r)r}$

$w_e=\frac{1}{2}\varepsilon_0\varepsilon_r(r)\frac{Q'^2}{4\pi^2(\varepsilon_0\varepsilon_r(r))^2 r^2}$

$w_e=\frac{1}{8}\frac{Q'^2}{\pi^2 r^2\varepsilon_0\varepsilon_r(r)} \qquad w_e=\frac{A}{r^2\varepsilon_r(r)}B$

$r^2\varepsilon_r(r)=C\ (A/B)$

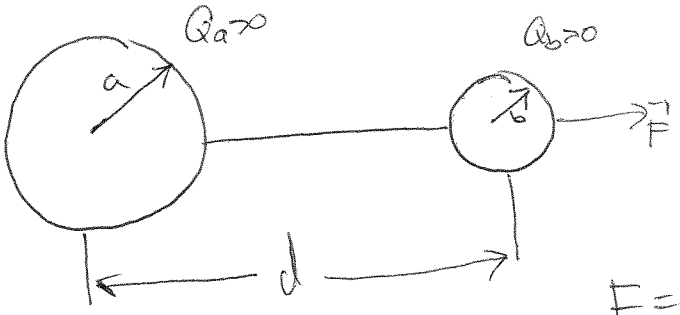
$r=a \rightarrow \varepsilon_r(a)=5$

$C=5a^2$

$\varepsilon_r(r)\cdot r^2=5a^2 \Rightarrow$ $\boxed{\varepsilon_r(r)=5\left(\frac{a}{r}\right)^2}$

4. децембар 2010

1. питомца



$V_a=V_b$

$Q_a+Q_b=Q$

$a,b<<d$

$V=\frac{Q}{4\pi\varepsilon_0 r}$

$F=\frac{Q_aQ_b}{4\pi\varepsilon_0 d^2}$

$\frac{Q_a}{4\pi\varepsilon_0 a}=\frac{Q_b}{4\pi\varepsilon_0 b}$

$Q_a=\frac{a}{b}Q_b$

$\frac{a}{b}Q_b+Q_b=Q$

$Q_b=\frac{b}{a+b}Q$

$Q_a+\frac{b}{a}Q_a=Q$

$Q_a=\frac{a}{a+b}Q$

$F=\frac{ab}{(a+b)^2}\frac{1}{4\pi\varepsilon_0 d^2}$

$\frac{dF}{da}=0$

$\frac{b(a+b)^2-2ab(a+b)}{(a+b)^4}=0$

$b(a^2+2ab+b^2)-2ab(a+b)=0$

$ba^2+2ab^2+b^3-2a^2b-2ab^2=0$

$b^2-a^2=0$

$\boxed{b=a}$

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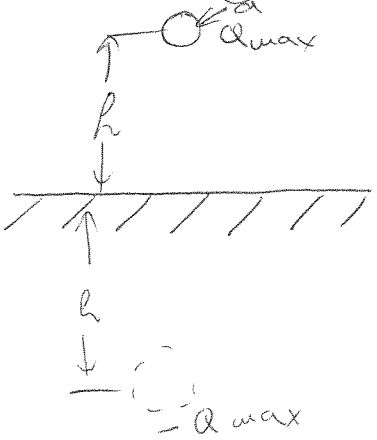
2. питање



$h \gg a$

E_{kr}

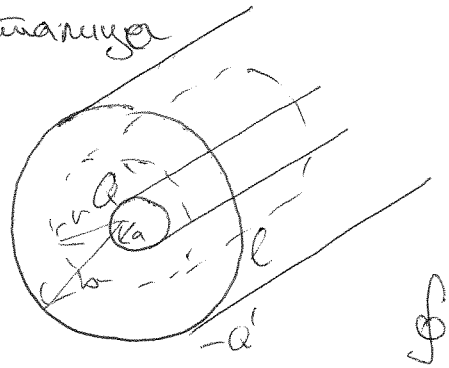
$$E_{kr} = \frac{Q_{max}}{4\pi\varepsilon_0 a^2} \Rightarrow Q_{max} = 4\pi\varepsilon_0 a^2 E_{kr}$$

$$F_{max} = \frac{Q_{max} \cdot Q_{max}}{4\pi\varepsilon_0 (2h)^2}$$

$$F_{max} = \frac{Q_{max}^2}{16\pi\varepsilon_0 h^2} = \frac{16\pi^2\varepsilon_0^2 a^4 E_{kr}^2}{16\pi\varepsilon_0 h^2}$$

$$\boxed{F_{max} = \pi\varepsilon_0 \frac{a^4}{h^2} E_{kr}^2}$$

3. питање



$a = 1\,mm$

$b \Leftarrow \left(\frac{b}{a} = e\right)$

$\varepsilon_r = \frac{5}{\pi}$

$$C' = \frac{Q'}{U}$$

$$U = \int_a^b E(r)\,dr$$

$$\oint \vec{D}\,d\vec{s} = Q_{vs}$$

$$D(r) \cdot 2\pi r \cdot l = Q' l$$

$$D(r) = \frac{Q'}{2\pi r}$$

$$D(r) = \varepsilon_0 \varepsilon_r E(r)$$

$$E(r) = \frac{Q'}{2\pi\varepsilon_0 \varepsilon_r r}, \quad \varepsilon_r = \frac{5}{\pi}$$

$$E(r) = \frac{Q'}{10\varepsilon_0 r}$$

$$U = \int_a^b \frac{Q'}{10\varepsilon_0 r} dr = \frac{Q'}{10\varepsilon_0} \int_a^b \frac{dr}{r} = \frac{Q'}{10\varepsilon_0} \ln\frac{b}{a}$$

$$C' = \frac{Q'}{\frac{Q'}{10\varepsilon_0} \ln\frac{b}{a}} = \frac{10\varepsilon_0}{\ln\frac{b}{a}} \Rightarrow \boxed{C' = 10\varepsilon_0}$$

($\ln\frac{b}{a} = 1$)

4. питање

$\varepsilon_r = 4$

$Q_p = 3\mu C$

$$Q_p = -Q\left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right) \Rightarrow Q = -Q_p \frac{\varepsilon_r}{\varepsilon_r - 1}$$

$$Q = -3\mu C \frac{4}{4-1} = -3\mu C \frac{4}{3}$$

$$\boxed{Q = -4\mu C}$$

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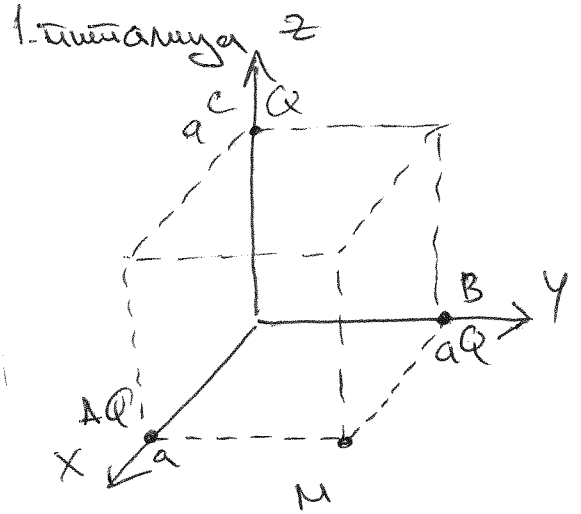
14. Новембар 2009

1. Питалица



$$\vec{E} = \vec{E}_A + \vec{E}_B + \vec{E}_C$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 a^2}\left(\left(1+\frac{\sqrt{3}}{9}\right)(\vec{1}_x+\vec{1}_y) - \frac{\sqrt{3}}{9}\vec{1}_z\right)}$$

Једнача: $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ – тачкасто наелектрисање

① тачка A

$r = a$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 a^2}\vec{1}_x$$

② тачка B

$r = a$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 a^2}\vec{1}_y$$

③ тачка C $\quad r = a\sqrt{3} \quad \cos\alpha = \frac{1}{\sqrt{3}}$

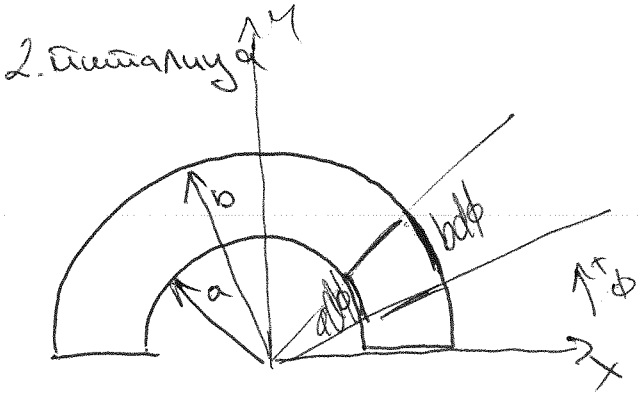
$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 3a^2} = \frac{Q}{12\pi\varepsilon_0 a^2}$$

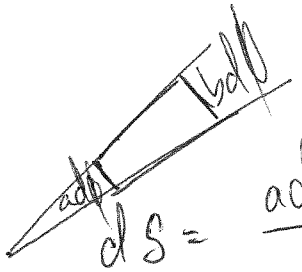
$$\vec{E}_z = -\frac{Q}{12\pi\varepsilon_0 a^2}\frac{1}{\sqrt{3}}\vec{1}_z$$

$$\vec{E}_x = \frac{Q}{12\pi\varepsilon_0 a^2}\frac{1}{\sqrt{3}}\vec{1}_x$$

$$\vec{E}_y = \frac{Q}{12\pi\varepsilon_0 a^2}\frac{1}{\sqrt{3}}\vec{1}_y$$

2. Питалица





$$dS = \frac{a\,d\phi + b\,d\phi}{2}(b-a) = \frac{b^2-a^2}{2}d\phi$$

СМЕНА: $\frac{\phi}{2} = t \quad d\phi = 2dt$

$$dQ = \rho_s\,dS$$

$$dQ = \rho_{s0}\cos\frac{\phi}{2}\frac{b^2-a^2}{2}d\phi$$

$$Q = \rho_{s0}\frac{b^2-a^2}{2}\int_0^{\pi}\cos\frac{\phi}{2}d\phi$$

$$= \rho_{s0}\frac{b^2-a^2}{2}\cdot 2\sin\frac{\phi}{2}\Big|_0^{\pi}$$

$$= \rho_{s0}(b^2-a^2)\left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$\boxed{Q = \rho_{s0}(b^2-a^2)}$$

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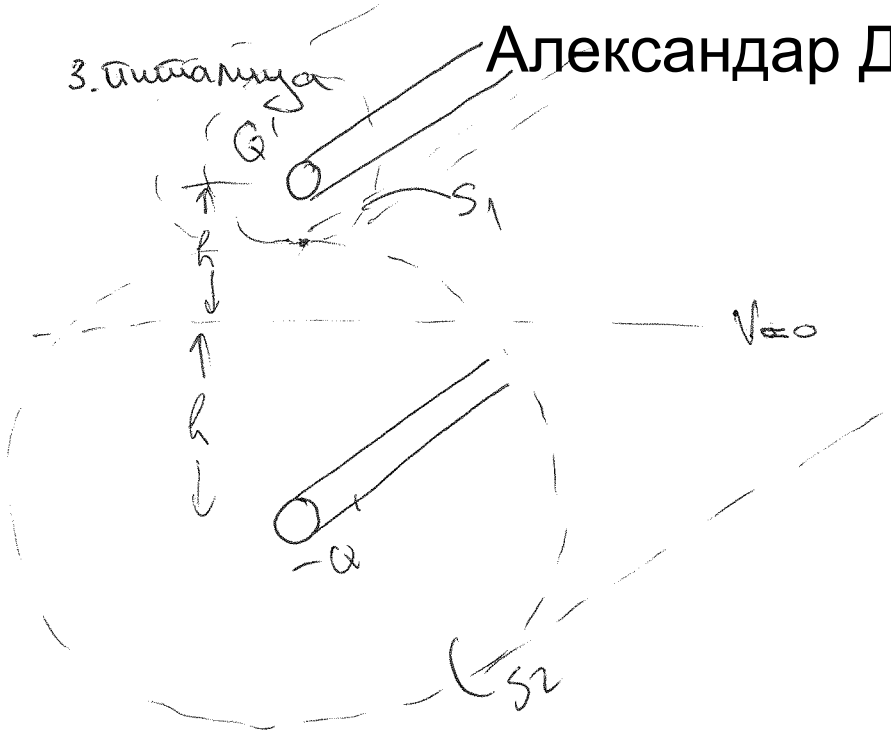
3. Питамнуа



$h/a > 10$

$h \gg a$

$a = 5\,mm$

$h = 10\,m$

$$C' = \frac{Q'}{U}$$

$$C' = \frac{Q'}{V - V_\infty}, \quad V_\infty = 0$$

$$V = \int_a^h \vec{E}_{uk}\, d\vec{h}$$

$$\oint_{S_1} E_a\, dS = \frac{Q_{uS_1}}{\varepsilon_0}$$

$$E_{a'} \cdot 2\pi r h = \frac{a' h}{\varepsilon_0}$$

$$E_{a'} = \frac{Q'}{2\pi\varepsilon_0 r}$$

Идентично за доњу воду (тик)

$$E_{-a'} = \frac{Q'}{2\pi\varepsilon_0 (2h - r)}$$

$$V = \frac{Q'}{2\pi\varepsilon_0} \int_a^h \left[\frac{1}{r} + \frac{1}{2h - r} \right] dh$$

. . . . . .

. . . . . среди се интеграл и добије се да је потенцијал једнак

$$V = \frac{a'}{2\pi\varepsilon_0} \ln \frac{2h}{a}$$

$$C' = \frac{Q'}{V} \Rightarrow C' = \frac{2\pi\varepsilon_0}{\ln \frac{2h}{a}} \Rightarrow \boxed{C' \approx 6{,}71\ pF/m}$$

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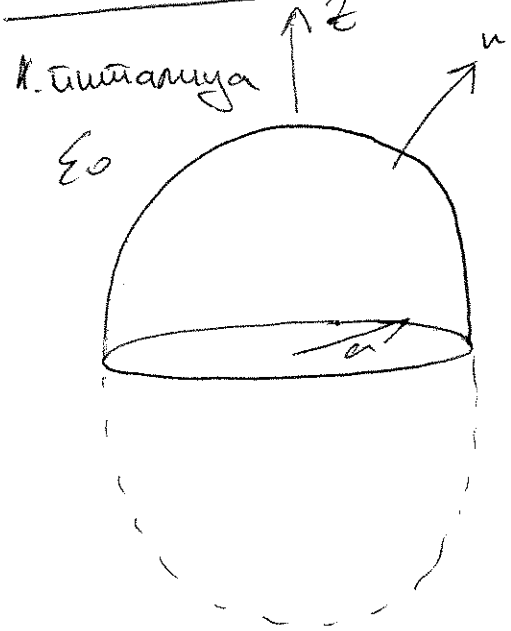
22. Новембар 2008

1. задатак



$$\oint_S E\,dS = \frac{Q_{unS}}{\varepsilon_0}$$

$$\Psi = \int_S \vec{E}\,d\vec{S} = \phi \text{ - флукс}$$

$$\oint_S E\,dS = \frac{Q}{\varepsilon_0}\,\Psi$$

$$\Downarrow$$

$$2\int_{S_1} E\,dS = \frac{Q}{\varepsilon_0}$$

$$\boxed{\Psi = \frac{Q}{2\varepsilon_0}}$$

2. задатак

$$\oint_S \vec{D}\,d\vec{S} = Q_{unS} = Q$$

$$-\oint_S P\,dS = Q_P$$

$$\vec{D} = \varepsilon\vec{E}$$

$$D = \varepsilon_0 E + P$$

$$D = \varepsilon_0 \frac{D}{\varepsilon} + P$$

$$D\left(1 - \frac{\varepsilon_0}{\varepsilon}\right) = P \quad / \oint_S dS$$

$$\oint_S D\,dS\left(\frac{\varepsilon - \varepsilon_0}{\varepsilon}\right) = \oint_S P\,dS$$

$$\boxed{Q_P = -Q\,\frac{\varepsilon - \varepsilon_0}{\varepsilon}}$$

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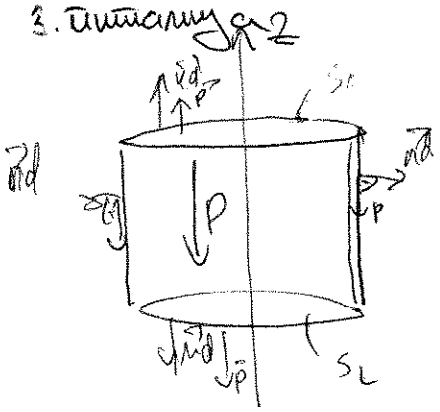
3. Питалица 2



$\vec{P} = -P_0 \vec{i}_z$

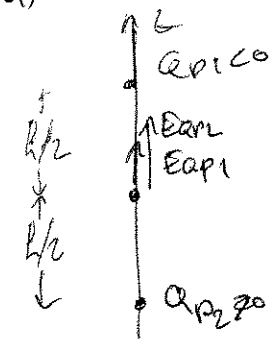
$\sigma_p = \vec{P}\vec{n_d}$

$\sigma_{\text{хоризонтално}} = 0 \quad \sphericalangle(\vec{P}, \vec{n_d}) = 90°$

$\sigma_{p1} = \vec{P}\vec{n_d} = -P_0 \vec{i}_z \vec{i}_z = -P_0 \Rightarrow Q_{p1} = -P_0 a^2 \pi$

$\sigma_{p2} = \vec{P}\vec{n_d} = -P_0 \vec{i}_z (-\vec{i}_z) = P_0 \Rightarrow Q_{p2} = P_0 a^2 \pi$

а) $h \gg a$

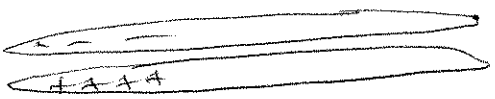


$E_S = 2 \frac{Q_{p2}}{4\pi\varepsilon_0 (h/2)^2} \vec{i}_z'$

$= 2 \frac{P_0 a^2 \pi}{4\pi\varepsilon_0 h^2} \vec{i}_z$

$\boxed{\vec{E}_S = \frac{2P_0 a^2}{\varepsilon_0 h^2} \vec{i}_z}$

б)



$\oint_S \vec{E} d\vec{s} = Q_{unS}/\varepsilon_0$

$E_S \Delta S + E_S \Delta S = \frac{1}{\varepsilon_0} \Delta S \sigma_{p2}$

$2E_S = \frac{\sigma_{p2}}{\varepsilon_0}$

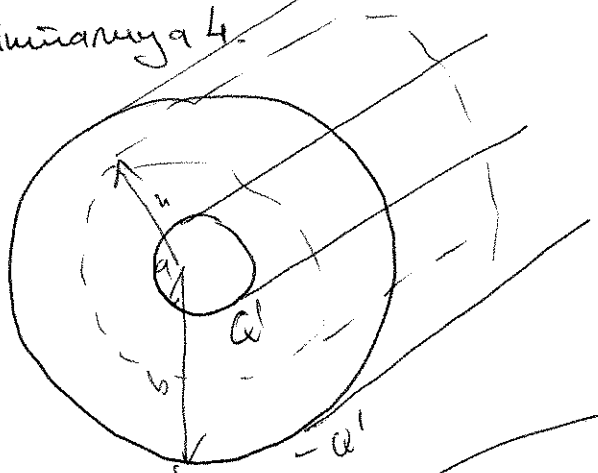
$\vec{E}_S = \frac{P_0}{2\varepsilon_0}$

$\vec{E}_{uk} = 2\vec{E}_S \vec{i}_z$

$\boxed{\vec{E}_{uk} = \frac{P_0}{\varepsilon_0} \vec{i}_z}$

$\varepsilon_r = 2{,}25 = \frac{9}{4} \quad E_{kr} = 50\ MV/m$

Питалица 4.



$\oint_S \vec{D} d\vec{s} = Q_{unS}$

$D(r) 2\pi r \cdot l = Q' \cdot l$

$D(r) = \frac{Q'}{2\pi r} \Rightarrow E(r) = \frac{Q'}{2\pi\varepsilon_0\varepsilon_r r}$

$U = \int_a^b \frac{Q'}{2\pi\varepsilon_0\varepsilon_r r} \Rightarrow U = \frac{Q'}{2\pi\varepsilon_0\varepsilon_r} \ln b/a$

$U_{kr} = E_{kr} a \ln b/a \qquad U_{max} = \frac{Q'}{2\pi\varepsilon_0\varepsilon_r}$

$E_{kr} = \frac{Q'_{max}}{2\pi\varepsilon_0\varepsilon_r a}$

$Q'_{max} = 2\pi\varepsilon_0\varepsilon_r a E_{kr}$

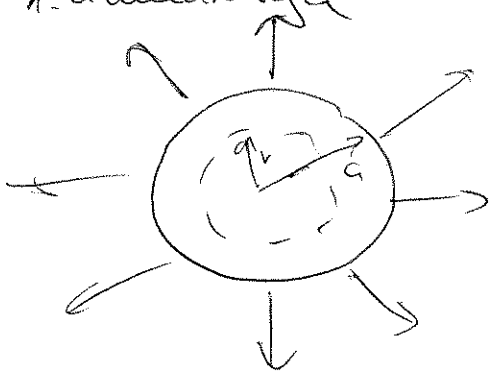
$U_{kr} = 50 \cdot 10^6\ V/m \cdot 1{,}84 \cdot 10^{-3}$ $\quad a \sim b/e$

$\Rightarrow \boxed{U_{kr} = 92\ kV/m}$

1. децембар - 2007

Александар Дацић

1. задатак



$a = 10\,mm$

$E_{kr} = 3\,MV/m$

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$V_{max} = \frac{Q_{max}}{4\pi\varepsilon_0 a}$$

$$V_{max} = \frac{4\pi\varepsilon_0 a^2 E_{kr}}{4\pi\varepsilon_0 a}$$

$$V_{max} = E_{kr}\, a$$

$$V_{max} = 30\,kV$$

$$\oint_S \vec{E}\,d\vec{S} = Q_{us}/\varepsilon_0$$

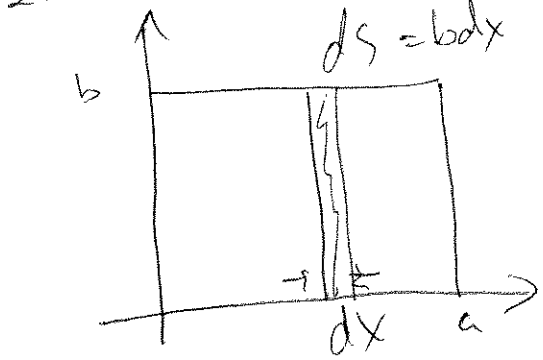
$$E(r)\cdot 4\pi r^2 = Q/\varepsilon_0$$

$$E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E_{kr} = \frac{Q_{max}}{4\cdot\pi\varepsilon_0 a^2}$$

$$Q_{max} = 4\pi\varepsilon_0 a^2 E_{kr} = 33{,}4\,nC$$

2. задатак



$$\rho_s(x) = \rho_{s0}\sin\frac{\pi x}{a}$$

$$dQ = \rho_s\, ds$$

$$Q = \int_0^a dQ = \int_0^a \rho_{s0}\sin\frac{\pi x}{a}\, b\,dx$$

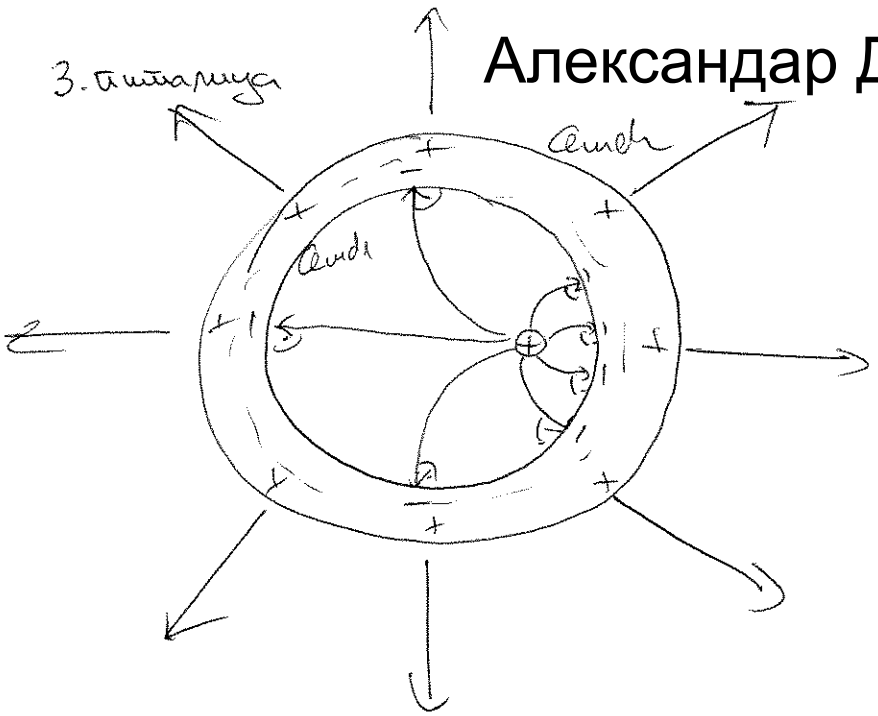
$$Q = \rho_{s0}\, b\,\frac{a}{\pi}\int \sin t\,dt$$

$$= \rho_{s0}\, b\,\frac{a}{\pi}\left(-\cos\frac{\pi x}{a}\right)\Big|_0^a$$

$$= \rho_{s0}\,\frac{ba}{\pi}\left(-\cos\pi + \cos 0^\circ\right)^2$$

$$Q = 2\rho_{s0}\,\frac{ba}{\pi}$$

3. питање



$$\oint_S \vec{D}\,d\vec{S} = \frac{Q_{us}}{\varepsilon_0}$$

$$0 = \frac{Q + Q_{ind1}}{\varepsilon_0}$$

$$Q_{ind1} = -Q$$

$$\sum_{i=1}^{n} Q_{ind1} = 0$$

$$Q_{ind1} + Q_{ind2} = 0$$

$$\boxed{Q_{ind2} = Q}$$

4. питање

$\varepsilon_r = 2$

$Q = 2\mu C$

$$\oint_S \vec{D}\,d\vec{S} = Q_{us} = Q$$

$$-\oint_S \vec{P}\,d\vec{S} = Q_P$$

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \vec{D} - \varepsilon_0 \frac{D}{\varepsilon_0 \varepsilon_r}$$

$$\vec{P} = \vec{D}\left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right) \Big/ \oint_S dS$$

$$-Q_P = Q\left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right)$$

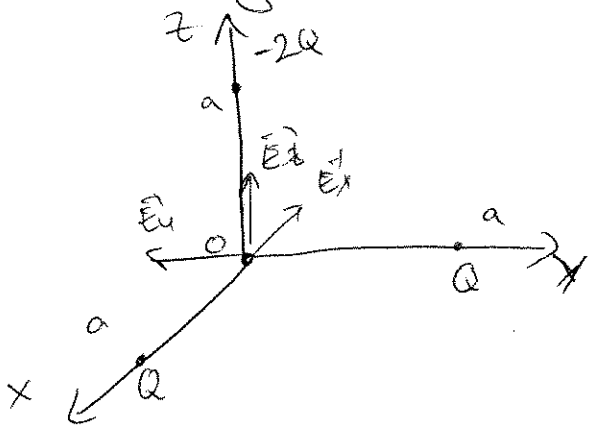
$$\boxed{Q_P = -Q\left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right)}$$

$$\boxed{Q_P = -1\mu C}$$

Александар Дацић

3. децембар 2006

1. питалица



Теорија: $V = \frac{Q}{4\pi\varepsilon_0 r}$

$E = \frac{Q}{4\pi\varepsilon_0 r^2}$

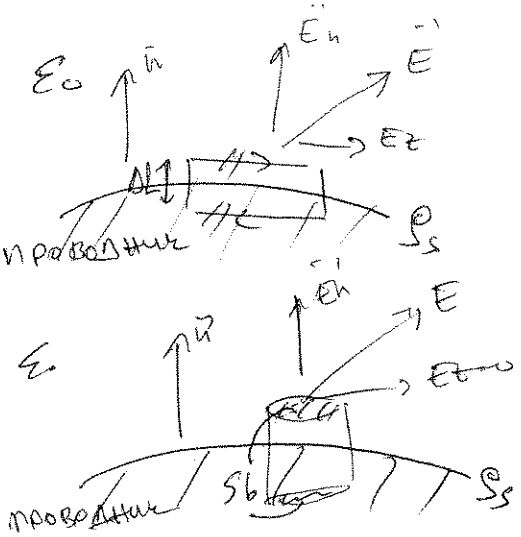
$V_O = \frac{Q}{4\pi\varepsilon_0 a} + \frac{Q}{4\pi\varepsilon_0 a} + \frac{-2Q}{4\pi\varepsilon_0 a}$

$$\boxed{V_O = 0}$$

$$\vec{E}_O = \frac{Q}{4\pi\varepsilon_0 a^2}\left(-\vec{\imath}_x - \vec{\imath}_y - (-2\vec{\imath}_z)\right)$$

$$\boxed{\vec{E}_O = \frac{Q}{4\pi\varepsilon_0 a^2}\left(-\vec{\imath}_x - \vec{\imath}_y + 2\vec{\imath}_z\right)}$$

2. питалица



$$\left.\begin{aligned}\oint_c \vec{E}\,d\vec{l} = 0\\ \Delta l \to 0 = 0\end{aligned}\right\} \quad \vec{E}_t = 0$$

$$\oint_s E\,ds = \frac{Q_{us}}{\varepsilon_0}$$

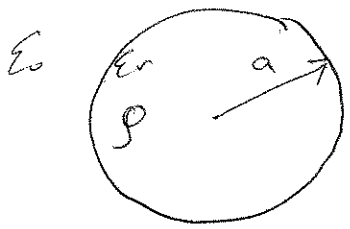
$\Delta h \to 0$

$$E_n \cdot Sb = Sb\,\frac{\rho_s}{\varepsilon_0}$$

$$\boxed{E_n = \frac{\rho_s}{\varepsilon_0}}$$

Александар Дачић

3. Питање



$r < a$

$$\oint_S \vec{D}\,d\vec{s} = \frac{Q_{us}}{\varepsilon_0}$$

$$D(r)\cdot 4r^2\pi = \frac{\rho \frac{4}{3} r^3 \pi}{\varepsilon_0}$$

$$D(r) = \frac{\rho r}{3\varepsilon_0}$$

$$\vec{D}(r) = \varepsilon_0 \varepsilon_r \vec{E}(r)$$

$r > a$

$$\oint_S \vec{D}\,d\vec{s} = \frac{Q_{us}}{\varepsilon_0}$$

$$D(r)\cdot 4r^2\pi = \frac{Q_{us}}{\varepsilon_0}$$

$$D(r) = 4r^2\pi = \frac{\rho \frac{4}{3} a^3 \pi}{\varepsilon_0}$$

$$D(r) = \frac{\rho a^3}{3\varepsilon_0 r^2}$$

$$\vec{D}(r) = \varepsilon_0 \vec{E}(r)$$

$$E(r) = \begin{cases} \dfrac{\rho r}{3\varepsilon_0\varepsilon_r}, & r < a \\ \dfrac{\rho a^3}{3\varepsilon_0 r^2}, & r > a \end{cases}$$

Теорија:

$$V = \int_{+\infty}^{} \vec{E}\,d\vec{l}$$

$$V = \int_0^a E(r)\,dr + \int_{a}^{+\infty} E(r)\,dr$$

$$= \int_0^a \frac{\rho r}{3\varepsilon_0\varepsilon_r}\,dr + \int_0^{+\infty} \frac{\rho a^3}{3\varepsilon_0 r^2}\,dr$$

$$= \frac{\rho}{3\varepsilon_0}\left(\frac{1}{\varepsilon_r}\int_0^a r\,dr + a^3\int_a^{+\infty} \frac{-1}{r^2}\,dr\right)$$

$$= \frac{\rho}{3\varepsilon_0}\left(\frac{1}{\varepsilon_r}\,\frac{r^2}{2}\Big|_0^a + a^3\left(-\frac{1}{r}\right)_a^{+\infty}\right)$$

$$= \frac{\rho}{3\varepsilon_0}\left(\frac{1}{\varepsilon_r}\,\frac{a^2}{2} + a^3\,\frac{-1}{+\infty}^{0} + a^3\frac{1}{a}\right)$$

$$\boxed{V = \frac{\rho a^2}{6\varepsilon_0}\left(\frac{1}{\varepsilon_r} + 2\right)}$$

Александар Дацић

4. питалица

$$\oint_S \vec{D}\,d\vec{S} = Q_{us}$$

$$D(r)\,4r^2\pi = Q$$

$$D(r) = \frac{Q}{4\pi r^2}$$

$$E(r) = \frac{Q}{4\pi\varepsilon r^2}$$

$$\vec{P} = \vec{D} - \varepsilon_0\vec{E}$$

$$= \frac{Q}{4\pi r^2}\left(1 - \frac{\varepsilon_0}{\varepsilon}\right) =$$

$$\vec{P} = \frac{Q}{4\pi r^2}\left(\frac{\varepsilon - \varepsilon_0}{\varepsilon}\right)$$

$$\sigma_{pa} = \vec{P}\vec{n} = -P(a)$$

$$\sigma_{pa} = -\frac{Q}{4\pi a^2}\left(\frac{\varepsilon - \varepsilon_0}{\varepsilon}\right)$$

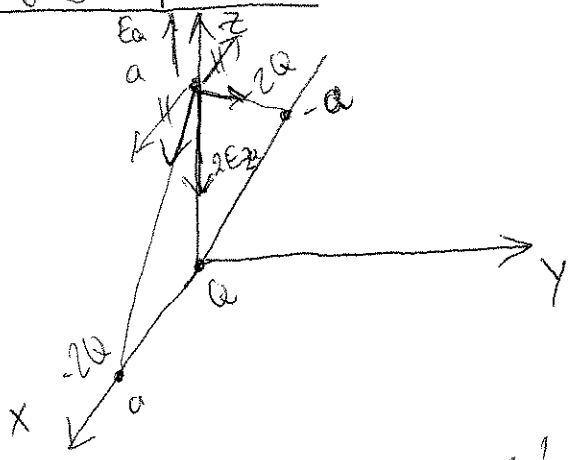
$$\boxed{Q_{pa} = -Q\,\frac{\varepsilon - \varepsilon_0}{\varepsilon}}$$

$$\sigma_{pb} = \vec{P}\vec{n} = P(b)$$

$$\sigma_{pb} = \frac{Q}{4\pi b^2}\,\frac{\varepsilon - \varepsilon_0}{\varepsilon}$$

$$\boxed{Q_{pb} = Q\left(\frac{\varepsilon - \varepsilon_0}{\varepsilon}\right)}$$

4. децембар 2005



$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \qquad \alpha = \pi/4$$

① и ③ $$E = \frac{2Q}{4\pi\varepsilon_0(a\sqrt{2})^2}$$

$$= \frac{2Q}{4\pi\varepsilon_0 2a^2}$$

$$\vec{E}_z = 2E\cos\alpha$$

$$= 2\frac{Q}{4\pi\varepsilon_0 a^2}\cos\pi/4\,(-\vec{i}_z)$$

$$\vec{E}_{z13} = -\frac{Q\sqrt{2}}{4\pi\varepsilon_0 a^2}\vec{i}_z$$

② $$\vec{E}_{z2} = \frac{Q}{4\pi\varepsilon_0 a^2}\vec{i}_z\cos 0^\circ$$

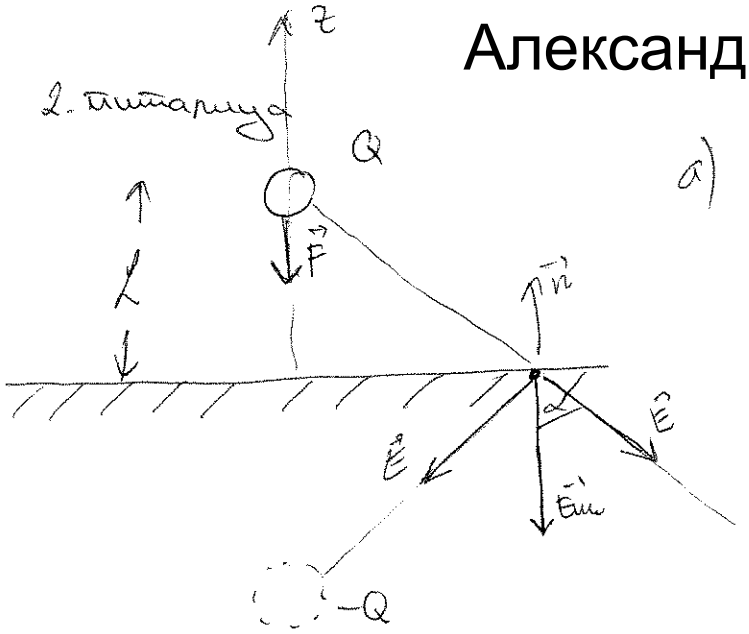
$$\vec{E}_{z2} = \frac{Q}{4\pi\varepsilon_0 a^2}\vec{i}_z \qquad \vec{E}_{uk} = \vec{E}_z = \vec{E}_{z13} + \vec{E}_{z2}$$

$$\vec{E}_z = \left(\frac{Q}{4\pi\varepsilon_0 a^2} - \frac{Q\sqrt{2}}{4\pi\varepsilon_0 a^2}\right)\vec{i}_z$$

$$\boxed{\vec{E}_z = \frac{Q}{4\pi\varepsilon_0 a^2}\left(1 - \sqrt{2}\right)\vec{i}_z}$$

Александар Дацић

2. питалица



a) $\vec{F} = Q\vec{E}_{-Q} = Q\frac{Q}{4\pi\varepsilon_0(2l)^2}(-\vec{1}_z)$

$$\boxed{\vec{F} = -\frac{Q^2}{16\pi\varepsilon_0 l^2}\vec{1}_z}$$

$\cos\alpha = \frac{l}{\sqrt{r^2+l^2}}$

$\rho_{ind} = \vec{D}\vec{n} = \varepsilon_0 E_{uk}\vec{n}$

$\rho_{ind} = -\varepsilon_0 E_{uk}$

$E_{uk} = 2\frac{Q}{2\pi\varepsilon_0(r^2+l^2)}\cdot\cos\alpha$

$E_{uk} = \frac{Q}{2\pi\varepsilon_0(r^2+l^2)^{3/2}}\cdot l$ $\quad r \to 0$

$E_{uk} = \frac{Ql}{2\pi\varepsilon_0 l^3} = \frac{Q}{2\pi\varepsilon_0 l^2}$

$\rho_{ind} = -\varepsilon_0 E_{uk}$

$= -\varepsilon_0 \frac{Q}{2\pi\varepsilon_0 l^2}$

$$\boxed{\rho_{ind} = \frac{Q}{2\pi l^2}}$$

3. питалица

$\oint_S \vec{D}d\vec{S} = Q_{us} = Q$

$-\oint_S \vec{P}d\vec{S} = Q_P$

$\vec{D} = \varepsilon\vec{E}$

$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$

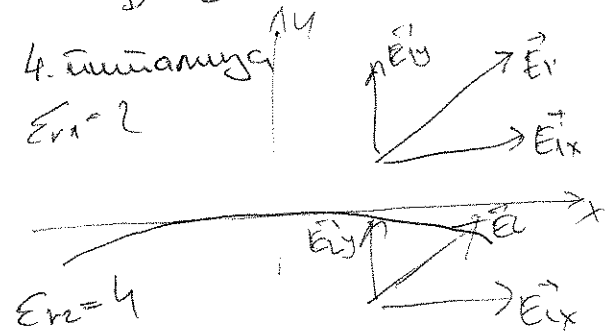
$\vec{P} = \vec{D} - \frac{\varepsilon_0}{\varepsilon}\vec{D} = \vec{D}\left(\frac{\varepsilon-\varepsilon_0}{\varepsilon}\right) \Big/ \oint_S dS$

$\oint_S \vec{P}d\vec{S} = \oint_S \vec{D}d\vec{S}\left(\frac{\varepsilon-\varepsilon_0}{\varepsilon}\right)$

$-Q_P = Q\left(\frac{\varepsilon-\varepsilon_0}{\varepsilon}\right)$

$Q = 0 \Rightarrow \boxed{Q_P = 0}$

4. питалица



$\varepsilon_{r1} = 2$

$\varepsilon_{r2} = 4$

$E_1 = 2\vec{1}_x + 4\vec{1}_y$ V/m

$E_{1x} = E_{2x} = 2$ V/m

$D_2 = \varepsilon_0\varepsilon_{r2}\vec{E}_2$

$\frac{tg\alpha_1}{tg\alpha_2} = \frac{2}{4} = \frac{1}{2} = \frac{E_{1x}/E_{1y}}{E_{2x}/E_{2y}} \Rightarrow E_{2y} = E_{1y}\frac{1}{2}$

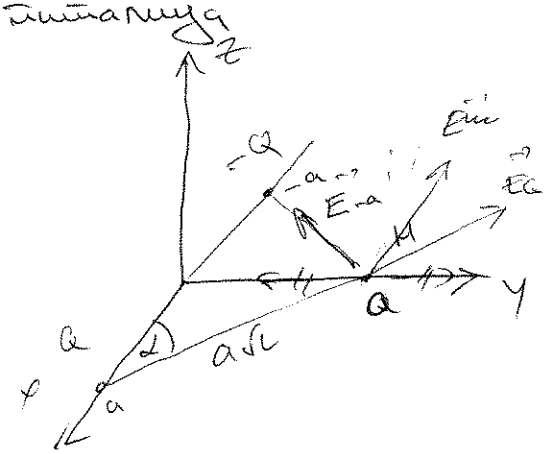
$\vec{E}_{2y} = 4\cdot\frac{1}{2} = 2$ V/m

$$\boxed{\vec{E}_2 = 2\vec{1}_x + 2\vec{1}_y \text{ V/m}}$$

28. Новембар 2004

Александар Дацић

1. питалица



Теорија: $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ $\alpha = \pi/4$

$$E_{uk} = 2\frac{Q}{4\pi\varepsilon_0 (a\sqrt{2})^2}\cos\alpha$$

$$= 2\frac{Q}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2}$$

$$E_{uk} = \frac{Q\sqrt{2}}{8\pi\varepsilon_0 a^2}$$

$$\boxed{\vec{E}_{uk} = -\frac{Q\sqrt{2}}{8\pi\varepsilon_0 a^2}\vec{i}_x}$$

2. питалица

а) ~~Л~~ А б) НЕ, јер се добије бесконачни потенцијал уоколини блакна.

3. питалица

① $\oint_C \vec{E}\,d\vec{l} = 0$ ② $\oint_S \varepsilon\vec{E}\,d\vec{S} = Q$

4. питалица

$\vec{D}\vec{n} = \sigma$

$\vec{\nabla}\vec{P} = \rho_p$

$\rho_p = -P$

$\sigma = D$

$D = \varepsilon_0 E + P = \varepsilon E$

$$P = \frac{\varepsilon - \varepsilon_0}{\varepsilon} D$$

$$\boxed{\rho_p = -\frac{\varepsilon - \varepsilon_0}{\varepsilon}\sigma}$$

Срећно на колоквијуму!!!