

Стефан Ђировић



ОСНОВИ ЕЛЕКТРОТЕХНИКЕ 2

• ПРОМЕНЉИВЕ СТРУЈЕ •

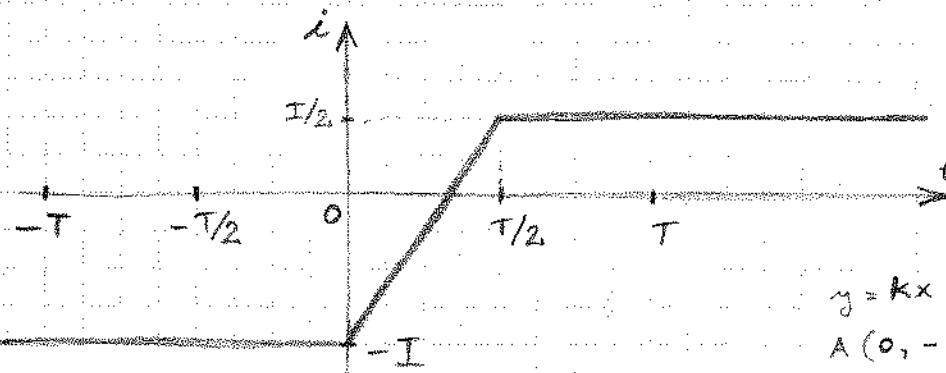
(РЕШЕНИ ЗАДАЦИ ИЗ ЗБИРКЕ I ДЕО)



Београд
2015.

1. ПРОМЕНЉИВЕ СТРУЈЕ

1.



$$y = kx + n$$

$$A(0, -I)$$

$$B(T/2, I/2)$$

$$-I = n \Rightarrow n = -I$$

$$\frac{I}{2} = k \frac{T}{2} + n = k \frac{T}{2} - I$$

$$\frac{3I}{2} = kT \Rightarrow k = \frac{3I}{T}$$

$$y = \frac{3I}{T} x - I$$

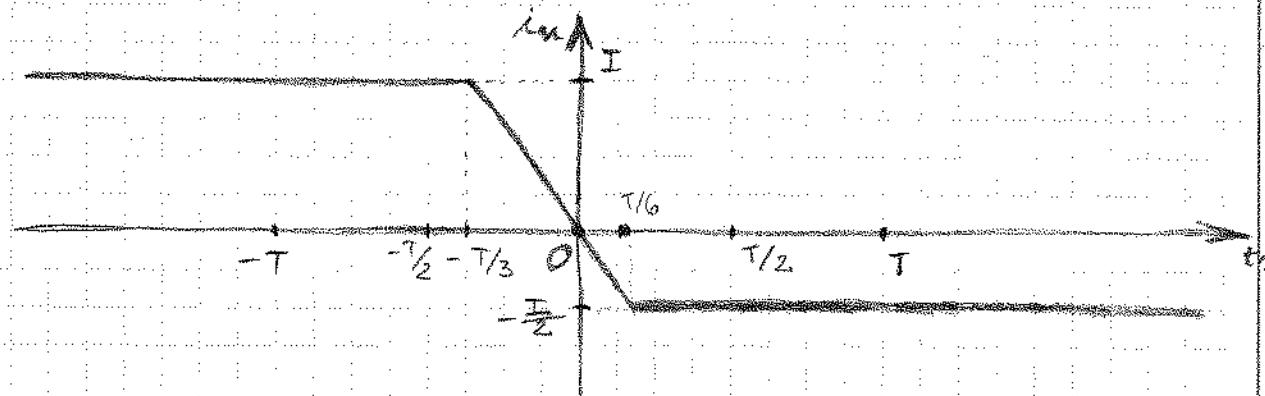
$$i(t) = \begin{cases} -I, & t \in (-\infty, 0) \\ \frac{3I}{T}t - I = \left(\frac{3}{T}t - 1\right)I, & t \in [0, \frac{T}{2}] \\ \frac{I}{2}, & t \in (\frac{T}{2}, +\infty) \end{cases}$$

5) МЕЊА СЕ РЕФЕРЕНТНИ СМЕР, А ПОЧЕТНИ ТРЕЧУТАК

ЈЕ ЗА $\frac{I}{3}$ КАСНЈЕ:

$$t_1 = t - \frac{I}{3} \Rightarrow t = t_1 + \frac{I}{3}$$

$$i_{n(t_1)} = \begin{cases} I, & t_1 \in (-\infty, -T/3) \\ -\left(\frac{3}{T}t_1 - 1\right)I = \left(1 - \frac{3}{T}t_1\right)I = \left(1 - \frac{3}{T}(t_1 + \frac{I}{3})\right)I = \left(1 - 1 - \frac{3I}{T}\right)I = -\frac{3I}{T}t_1, & t_1 \in [-T/3, T/6] \\ -\frac{I}{2}, & t_1 \in (T/6, +\infty) \end{cases}$$



$$2. i(t) = I_m |\cos \omega t| \quad I_m > 0$$

→ написати віспа з за супотах перенитні смр

у почтніх трансфера за $\frac{\pi}{3\omega}$ рахунок

$$t_1 = t + \frac{\pi}{3\omega} \Rightarrow t = t_1 - \frac{\pi}{3\omega}$$

$$i_m(t_1) = -I_m |\cos(\omega(t_1 - \frac{\pi}{3\omega}))| = -I_m |\cos(\omega t_1 - \frac{\pi}{3})|$$

$$\boxed{i_m(t_1) = -I_m |\cos(\omega t_1 - \frac{\pi}{3})|}$$

$$3. a) i(t) = 2 \sin \omega t \text{ A}$$

$$\sin \alpha = \cos(\frac{\pi}{2} - \alpha) = \cos(\alpha - \frac{\pi}{2})$$

$$i(t) = 2 \cos(\omega t - \frac{\pi}{2})$$

$$I_m = 2 \text{ A}, \quad I = \sqrt{2} \text{ A}, \quad \Psi = -\frac{\pi}{2}$$

$$d) u(t) = -15\sqrt{2} \cos(\omega t + \frac{\pi}{4}) \text{ V}$$

$$-\cos \alpha = \cos(\alpha \pm \pi)$$

$$u(t) = 15\sqrt{2} \cos(\omega t - \frac{3\pi}{4}) \text{ V}$$

$$b) e(t) = -\sin(\omega t + \frac{2\pi}{3}) \text{ mV}$$

$$-\sin \alpha = \cos(\alpha + \frac{\pi}{2})$$

$$e(t) = \cos(\omega t + \frac{2\pi}{3} + \frac{\pi}{2}) = \cos(\omega t + \frac{7\pi}{6}) \text{ mV}$$

$$\Rightarrow \boxed{e(t) = \cos(\omega t - \frac{5\pi}{6}) \text{ mV}}$$

$$E_m = 1 \text{ V} \quad E = \frac{\sqrt{2}}{2} \text{ mV}$$

$$u(t) = U_m \cos(\omega t + \Phi)$$

$$u(t) = U\sqrt{2} \cos(\omega t + \Phi)$$

$$U_m = U\sqrt{2} \quad (U_m > 0)$$

$$U = \frac{U_m}{\sqrt{2}} = U_m \frac{\sqrt{2}}{2} \quad (U > 0)$$

$\omega \rightarrow$ кривина часності (ω_0)

$\Phi \rightarrow$ почтна фаза ($-\pi \leq \Phi \leq \pi$)

$$T = \frac{2\pi}{\omega}, \quad f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$4. \quad i(t) = I_m \cos(\omega t + \Psi) = 6 \cos(100\pi t - \frac{\pi}{3}) A$$

$$i(t_1) = I_m \cos(\omega t_1 + \Psi_1) = 6 \cos(100\pi t_1 + \frac{7\pi}{6}) A$$

$$t = t_1 + \Delta t$$

$$i = I_m \cos(\omega t + \Psi) = I_m \cos(\omega(t_1 + \Delta t) + \Psi)$$

$$i = I_m \cos(\omega t_1 + \omega \Delta t + \Psi) = I_m \cos(\omega t_1 + (\omega \Delta t + \Psi))$$

$$i = I_m \cos(\omega t_1 + \Psi_1)$$

$$\Psi_1 = \omega \Delta t + \Psi \Rightarrow \omega \Delta t = \Psi_1 - \Psi$$

$$\Delta t = \frac{\Psi_1 - \Psi}{\omega}$$

$$\omega = 100\pi \text{ s}^{-1}$$

$$\Delta t = \frac{\frac{7\pi}{6} + \frac{\pi}{3}}{100\pi}$$

$$*5. \quad i_1(t) = I_{1m} \cos(\omega t + \Psi_1)$$

$$i_2(t) = I_{2m} \cos(\omega t + \Psi_2)$$

$$i_1(t) + i_2(t) = i(t) \Rightarrow i(t) = I_m \cos(\omega t + \Psi)$$

$$\Rightarrow I_{1m} \cos(\omega t + \Psi_1) + I_{2m} \cos(\omega t + \Psi_2) = I_m \cos(\omega t + \Psi)$$

$$I_{1m} (\cos \omega t \cdot \cos \Psi_1 - \sin \omega t \cdot \sin \Psi_1) + I_{2m} (\cos \omega t \cdot \cos \Psi_2 - \sin \omega t \cdot \sin \Psi_2)$$

$$= I_m (\cos \omega t \cdot \cos \Psi - \sin \omega t \cdot \sin \Psi)$$

$$\cos \omega t \cdot (I_{1m} \cdot \cos \Psi_1 + I_{2m} \cdot \cos \Psi_2) - \sin \omega t \cdot (I_{1m} \sin \Psi_1 + I_{2m} \sin \Psi_2)$$

$$= \cos \omega t \cdot (I_m \cdot \cos \Psi) - \sin \omega t \cdot (I_m \cdot \sin \Psi)$$

$$\Rightarrow (1) I_{1m} \cdot \cos \Psi_1 + I_{2m} \cdot \cos \Psi_2 = I_m \cdot \cos \Psi \quad \left. \right\}^2$$

$$(2) I_{1m} \cdot \sin \Psi_1 + I_{2m} \cdot \sin \Psi_2 = I_m \cdot \sin \Psi \quad \left. \right\}^2$$

$$I_{1m}^2 (\sin^2 \Psi_1 + \cos^2 \Psi_1) + I_{2m}^2 (\sin^2 \Psi_2 + \cos^2 \Psi_2) + 2 I_{1m} I_{2m} (\cos \Psi_1 \cos \Psi_2 + \sin \Psi_1 \sin \Psi_2) = I_m^2 (\sin^2 \Psi + \cos^2 \Psi)$$

$$I_{1m}^2 + I_{2m}^2 + 2I_{1m}I_{2m} \cos(\Psi_1 - \Psi_2) = I_m^2$$

$$I_m = \sqrt{I_{1m}^2 + I_{2m}^2 + 2I_{1m}I_{2m} \cos(\Psi_1 - \Psi_2)}$$

$I_m \geq 0$

$$\cos \Psi = \frac{I_{1m} \cos \Psi_1 + I_{2m} \cos \Psi_2}{I_m}$$

$$\sin \Psi = \frac{I_{1m} \sin \Psi_1 + I_{2m} \sin \Psi_2}{I_m}$$

$$\operatorname{tg} \Psi = \frac{\sin \Psi}{\cos \Psi} = \frac{I_{1m} \sin \Psi_1 + I_{2m} \sin \Psi_2}{I_{1m} \cos \Psi_1 + I_{2m} \cos \Psi_2} = \frac{I_{1m} \cdot \sin \Psi_1 + I_{2m} \cdot \sin \Psi_2}{I_{1m} \cdot \cos \Psi_1 + I_{2m} \cdot \cos \Psi_2}$$

$$\Psi = \begin{cases} \operatorname{arctg} \frac{I_{1m} \sin \Psi_1 + I_{2m} \sin \Psi_2}{I_{1m} \cos \Psi_1 + I_{2m} \cos \Psi_2}, & I_{1m} \cos \Psi_1 + I_{2m} \cos \Psi_2 > 0 \\ \frac{\pi}{2} \operatorname{sgn}(I_{1m} \cos \Psi_1 + I_{2m} \cos \Psi_2), & I_{1m} \cos \Psi_1 + I_{2m} \cos \Psi_2 = 0 \\ \operatorname{arctg} \frac{I_{1m} \sin \Psi_1 + I_{2m} \sin \Psi_2 + \pi \operatorname{sgn}(I_{1m} \sin \Psi_1 + I_{2m} \sin \Psi_2)}{I_{1m} \cos \Psi_1 + I_{2m} \cos \Psi_2}, & I_{1m} \cos \Psi_1 + I_{2m} \cos \Psi_2 < 0 \end{cases}$$

$$6. u(t) = u_1(t) + u_2(t) \quad U_1 > 0 \wedge U_2 > 0$$

$$u_1(t) = \sqrt{2} U_1 \sin \omega t = \sqrt{2} U_1 \cos \left(\omega t - \frac{\pi}{2} \right)$$

$$u_2(t) = \sqrt{2} U_2 \cos \omega t$$

$$u(t) = U_m \cos(\omega t + \theta) = U \sqrt{2} \cos(\omega t + \theta)$$

$$u(t) = \sqrt{2} U_1 \sin \omega t + \sqrt{2} U_2 \cos \omega t$$

$$u(t) = \sqrt{2} (U_1 \cos(\omega t - \frac{\pi}{2}) + U_2 \cos \omega t)$$

$$\text{ЗА } u_1(t) : U_{\text{eff}} = U_1 \quad \theta_1 = -\frac{\pi}{2}$$

$$\text{ЗА } u_2(t) : U_{\text{eff}} = U_2 \quad \theta_2 = 0$$

на схеме

$$\text{НЕТХОДИМЫЕ УДАРЫ: } U = \sqrt{U_1^2 + U_2^2 + 2U_1U_2 \cos(\theta_1 - \theta_2)} = \sqrt{U_1^2 + U_2^2}$$

$$\theta = -\operatorname{arctg} \frac{U_1}{U_2}$$

7. $u(t) = u_1(t) + u_2(t)$ $U_1 < 0 \wedge U_2 < 0$

$$u_1(t) = \sqrt{2} |U_1| \sin \omega t$$

$$u_2(t) = \sqrt{2} |U_2| \cos \omega t$$

канонични облици:

$$u_1(t) = \sqrt{2} |U_1| \cos \left(\omega t + \frac{\pi}{2} \right)$$

$$u_2(t) = \sqrt{2} |U_2| \cos (\omega t + \pi)$$

$$U = \sqrt{U_1^2 + U_2^2 + 2 U_1 U_2 \cos \left(\frac{\pi}{2} - \pi \right)} \Rightarrow U = \sqrt{U_1^2 + U_2^2}$$

$$\theta = -\arctg \left[\frac{|U_1|}{|U_2|} + i \right], \quad \text{да е } \quad 0 \leq \arctg \frac{|U_1|}{|U_2|} < \frac{\pi}{2} \quad \text{и } \frac{|U_1|}{|U_2|} >$$

8. $u(t) = (20 \sin \omega t + 20\sqrt{3} \cos \omega t) \vee$

$$u(t) = 20 \cos \left(\omega t - \frac{\pi}{2} \right) + 20\sqrt{3} \cos \omega t$$

$$u(t) = U_m \cos (\omega t + \theta)$$

$$\Rightarrow U_m = \sqrt{U_{1m}^2 + U_{2m}^2 + 2 U_{1m} U_{2m} \cos (\theta_1 - \theta_2)}$$

$$U_m = \sqrt{400 + 400 \cdot 3 + 2 \cdot 20 \cdot 20\sqrt{3} \cdot \cos \left(-\frac{\pi}{2} - 0 \right)} = \sqrt{4 \cdot 900} = 60 \text{ V}$$

$$U_m = 2 \cdot 20 \text{ V} \Rightarrow \boxed{U_m = 40 \text{ V}}$$

$$\theta = -\arctg \frac{U_{1m}}{U_{2m}} = -\arctg \frac{20}{20\sqrt{3}} = -\arctg \frac{1}{\sqrt{3}}$$

$$\boxed{\theta = -\frac{\pi}{6}}$$



9. $u(t) = U \cdot \cos \omega t + U\sqrt{2} \left(\cos \left(\omega t + \frac{3\pi}{4} \right) + \sin \left(\omega t - \frac{5\pi}{6} \right) \right)$

$$u(t) = U \cdot \cos \omega t + U\sqrt{2} \left(\cos \omega t \cdot \cos \frac{3\pi}{4} - \sin \omega t \cdot \sin \frac{3\pi}{4} \right) \\ + U \left(\cos \omega t \cdot \cos \frac{5\pi}{6} + \sin \omega t \cdot \sin \frac{5\pi}{6} \right)$$

$$u(t) = U \cos \omega t + U\sqrt{2} \cos \omega t \left(-\frac{\sqrt{3}}{2} \right) - U\sqrt{2} \sin \omega t \left(\frac{1}{2} \right) \\ U \cos \omega t \cdot \left(-\frac{\sqrt{3}}{2} \right) + U \sin \omega t \left(\frac{1}{2} \right)$$

$$u(t) = U \cos \omega t - U \cos \omega t - \frac{1}{2} U \sin \omega t - \frac{\sqrt{3}}{2} U \cos \omega t \\ \cos \left(\omega t - \frac{\pi}{2} \right)$$

$$u(t) = U_m \cos (\omega t)$$

$$U_m = \sqrt{\frac{1}{4} U^2 + \frac{3}{4} U^2 + 2 U^2 \sqrt{3} \cos 70^\circ \cos \left(-\frac{\pi}{2} - \theta \right)} = \sqrt{U^2}$$

$$\Rightarrow \boxed{U_m = U}$$

$$\theta = -\arctg \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} \Rightarrow \boxed{\theta = \frac{5\pi}{6}} \quad \left(\theta = -\frac{\pi}{6} \right)$$

10.

$$i(t) = I_m |\cos \omega t| \quad I_m > 0, \omega > 0, I_m, \omega = \text{const}$$

испитати ПЕРИОДИЧНОСТ СТРУJE:

$i(t)$ JE ПЕРИОДИЧНА АКО постоји $T \geq 0$

$$i(t) = i(t+T) \quad \forall t$$

$$I_m |\cos \omega t| = I_m |\cos (\omega(t+T))|$$

$$\omega T = k\pi, \quad k \in \mathbb{N} \quad k = 1, 2, 3, \dots$$

ОСНОВА:

ПЕРИОД :

$$\boxed{T = \frac{k\pi}{\omega}}$$

11. $i(t) = I_m \cos^3(\omega t + \varphi)$

$$i(t) = i(t+T)$$

$$I_m \cos^3(\omega t + \varphi) = I_m \cos^3(\omega t + \omega T + \varphi)$$

$$\omega t + \varphi = \omega t + \omega T + \varphi$$

$$\Rightarrow \omega T = 2k\pi, \quad k \in \mathbb{N}, \quad k=1, 2, 3, \dots$$

ОСНОВНИ
НЕРІОД:

$$T = \frac{2\pi}{\omega}$$

12. $i(t) = I \cos(\omega t) + \frac{I}{\pi} \cos(\pi \omega t)$

$$I \cos(\omega t) + \frac{I}{\pi} \cos(\pi \omega t) = I \cos(\omega t + \omega T) + \frac{I}{\pi} \cos(\pi \omega t + \pi \omega T)$$

$$\omega T = 2k_1\pi, \quad \wedge \quad \pi \omega T = 2k_2\pi \Rightarrow \omega T = 2k_2, \quad k_1, k_2 \in \mathbb{N}$$

$$T = \frac{2k_1\pi}{\omega}$$

$$k_2 = \frac{\omega T}{2}$$

$$k_1 = \frac{\omega T}{2\pi}$$

\nwarrow
 $k_1 \in \mathbb{N}$

13. испытать периодичность и основной период:

$$u(t) = \frac{U}{2\pi} \left(1 + \cos \omega t - \frac{1}{3} \cos 2\omega t \right)$$

$$u(t) = u(t+T)$$

$$\frac{U}{2\pi} \left(1 + \cos \omega t - \frac{1}{3} \cos 2\omega t \right) = \frac{U}{2\pi} \left(1 + \cos (\omega t + \omega T) - \frac{1}{3} \cos (2\omega t + 2\omega T) \right)$$

$$1 + \cos \omega t - \frac{1}{3} \cos 2\omega t = 1 + \cos (\omega t + \omega T) - \frac{1}{3} \cos (2\omega t + 2\omega T)$$

$$\omega T = 2k_1\pi \quad \wedge \quad 2\omega T = 2k_2\pi \Rightarrow \omega T = k_2\pi, \quad k_1, k_2 \in \mathbb{N}$$

$$\left(T = \frac{2k_1\pi}{\omega} = \frac{k_2\pi}{\omega} \right) \Rightarrow 2k_1 = k_2 \rightarrow k_2 \text{ парный}$$

основной период: $k_1=1, k_2=2 \rightarrow T = \frac{2\pi}{\omega}$

однажды частота (частота колебаний)

14. $i(t) = I_m \cos^2(\omega t + \Psi)$

$$i(t) = i(t+T)$$

$$I_m \cos^2(\omega t + \Psi) = I_m \cos^2(\omega t + \omega T + \Psi)$$

$$\omega T = k\pi$$

$$T = \frac{k\pi}{\omega} \rightarrow \text{основной период: } T = \frac{\pi}{\omega}$$

$$f = \frac{1}{T} \left(\frac{N}{t} \text{ где } N=1 \text{ период цикла} \right)$$

$$f = \frac{1}{\frac{\pi}{\omega}} \Rightarrow f = \frac{\omega}{\pi}$$

$$15. \quad u(t) = U \cos \omega t + \frac{U}{2} \cos \sqrt{2} \omega t + \frac{U}{3} \cos \sqrt{3} \omega t$$

$$u(t) = u(t+T)$$

$$U \cos \omega t + \frac{U}{2} \cos \sqrt{2} \omega t + \frac{U}{3} \cos \sqrt{3} \omega t = U \cos(\omega t + \omega T)$$

$$+ \frac{U}{2} \cos(\sqrt{2} \omega t + \sqrt{2} \omega T) + \frac{U}{3} \cos(\sqrt{3} \omega t + \sqrt{3} \omega T)$$

$$\omega T = 2k_1\pi \quad \wedge \quad \sqrt{2}\omega T = 2k_2\pi \quad \wedge \quad \sqrt{3}\omega T = 2k_3\pi, \quad k_1, k_2, k_3 \in \mathbb{N}$$

$$T = \frac{2k_1\pi}{\omega} = \frac{2k_2\pi}{\sqrt{2}\omega} = \frac{2k_3\pi}{\sqrt{3}\omega} \Rightarrow k_1 = \frac{k_2}{\sqrt{2}} = \frac{k_3}{\sqrt{3}}$$

ИАОН: КУЈЕ ПЕРИОДИЧАК!

$$16. \quad i(t) = I_0(|\cos \omega t| + A|\sin \omega t|), \quad A \neq 1$$

$$i(t) = I_0|\cos \omega t| + I_0A|\sin \omega t|$$

$$i(t) = i_1(t) + i_2(t)$$

$$i_1(t) = I_0|\cos \omega t|, \quad i_2(t) = I_0A|\sin \omega t|$$

$$i_1(t) = i_1(t+T) \quad i_2(t) = i_2(t+T)$$

$$I_0|\cos \omega t| = I_0|\cos(\omega t + \omega T)| \quad I_0A|\sin \omega t| = I_0A|\sin(\omega t + \omega T)|$$

$$\omega T = k_1\pi$$

$$\omega T = k_2\pi$$

о.н.: $T = \frac{\pi}{\omega}$

о.н.: $T = \frac{\pi}{\omega}$

ПЕРИОД ОБЕ СТРУКЕ ЈЕ ИСТА \rightarrow период $i(t)$: $T = \frac{\pi}{\omega}$

$$17. u(t) = U_1 \cos(12\pi f t) + U_2 \sin(18\pi f t)$$

$$u(t) = u_1(t) + u_2(t)$$

$$u_1(t) = U_1 \cos(12\pi f t) \quad u_2(t) = U_2 \sin(18\pi f t)$$

$$u_1(t) = u_1(t + T_1)$$

$$u_2(t) = u_2(t + T_2)$$

$$U_1 \cos(6\omega t) = U_1 \cos(6\omega t + 6\omega T_1) \quad U_2 \sin(9\omega t) = U_2 \sin(9\omega t + 9\omega T_2)$$

$$3\omega T_1 = 2k_1\pi \quad 9\omega T_2 = 2k_2\pi$$

$$3\omega T_1 = k_1\pi$$

$$9\omega T_2 = 2k_2\pi$$

$$6\pi T_1 f = k_1\pi$$

$$T_2 = \frac{1}{9f}$$

$$T_1 = \frac{1}{6f}$$

$$f_2 = 9f$$

$$\frac{T_1}{T_2} = \frac{\frac{1}{6f}}{\frac{1}{9f}} = \frac{3}{2} = \frac{3}{2} \in \mathbb{Q} \rightarrow \text{ПЕРИОДИЧНА ФУНКЦІЯ САМЕРНІВИ}$$

$\Rightarrow u(t)$ ПЕРИОДИЧНА ФУНКЦІЯ

$$T = n_1 T_1 = n_2 T_2, \quad n_1, n_2 \in \mathbb{N}$$

$$T = \frac{n_1}{6f} = \frac{n_2}{9f} \Leftrightarrow \frac{n_1}{6} = \frac{n_2}{9}, \quad \text{H3C}(6, 9) = 18 = k$$

$$9n_1 = 6n_2 = n$$

$$n_1 = \frac{n}{9} = \frac{18}{9} = \Rightarrow n_1 = 2$$

$$n_2 = \frac{n}{6} = \frac{18}{6} = \Rightarrow n_2 = 3$$

$$T = \frac{2}{6f} = \frac{3}{9f} \Rightarrow$$

$$T = \frac{1}{3f}$$

ОСНОВНИЙ

ПЕРИОД НАПОНА $u(t)$

УЧЕСТАНОСТЬ (ФРЕКВЕНЦІЯ) НАПОНА $u(t) : f = 3f \quad (f = \frac{1}{T})$

$$f = \text{H3D}(f_1, f_2) = 3f$$

$$6f - 9f$$

**

18.

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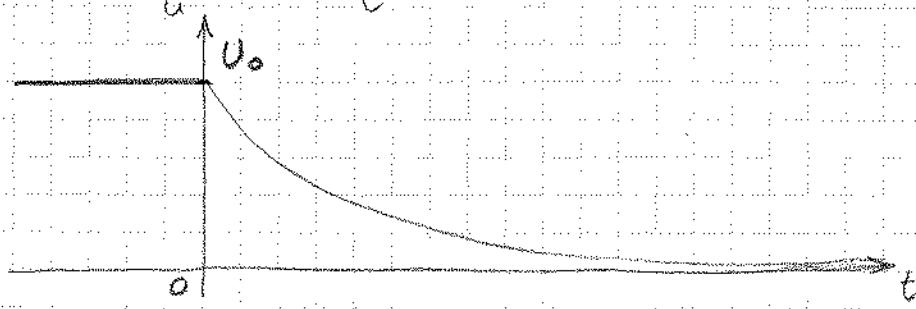
19.

ДА ли је функција периодична?

20.

a)

$$u(t) = \begin{cases} U_0, & t < 0 \\ U_0 e^{-\frac{t}{T}}, & t \geq 0 \end{cases}$$



ФУНКЦИЈА ЈЕ

АПЕРИОДИЧНА!

d) $u(t) = U_0 (e^{j\omega t} + e^{-j\omega t})$

$$u(t) = U_0 (\cos \omega t + j \sin \omega t + \cos \omega t - j \sin \omega t)$$

$u(t) = 2U_0 \cos \omega t$

$$u(t) = u(t+T)$$

ФУНКЦИЈА ЈЕ

$$2U_0 \cos \omega t = 2U_0 \cos(\omega t + \omega T)$$

ПЕРИОДИЧНА!

$$\omega T = 2k\pi$$

$T = \frac{2\pi}{\omega}$ → ОСНОВНИ
ПЕРИОД

b) $u(t) = U_0 (\sin \omega t + |\cos 2\omega t|)$

$$u(t) = u(t+T)$$

$$U_0 (\sin \omega t + |\cos 2\omega t|) = U_0 (\sin(\omega t + \omega T) + |\cos(2\omega t + 2\omega T)|)$$

$$\omega T = 2k_1\pi \quad \wedge \quad 2\omega T = k_2\pi$$

$T = \frac{2k_1\pi}{\omega}$

$T = \frac{k_2\pi}{2\omega}$

$$T = \frac{2k_1\pi}{\omega} = \frac{k_2\pi}{2\omega} \Rightarrow 2k_1 = \frac{k_2}{2} \Rightarrow (k_2 = 4k_1)$$

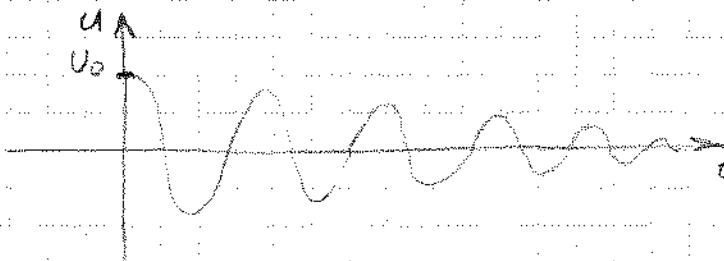
\Rightarrow ОСНОВНИ ПЕРИОД: $k_1=1, k_2=4 \rightarrow T = \frac{2\pi}{\omega}$

$T = \frac{2\pi}{\omega}$

ФУНКЦИЈА ЈЕ ПЕРИОДИЧНА!

i) $u(t) = U_0 e^{-dt} \cos(\omega t + \theta)$

1° Ako JE $\alpha > 0 \quad \theta = 0$



ФУНКЦИЈА ЈЕ

АЛЕРИОДИЧНА!

2° Ako JE $\alpha = 0$

$$u(t) = U_0 \cos(\omega t + \theta)$$

$$u(t) = u(t+T)$$

$$U_0 \cos(\omega t + \theta) = U_0 \cos(\omega t + \omega T + \theta)$$

$$\omega T = 2k\pi$$

$$T = \frac{2\pi}{\omega}$$

основни
период

ФУНКЦИЈА ЈЕ

ПЕРИОДИЧНА!

* 2.1. $U_m e^{-dt} \cos(\omega t + \theta) = \operatorname{Re}(\underline{\zeta})$ $\underline{\zeta} \rightarrow$ КОМПЛЕКСАЧ
БРОЈ

$$\underline{\zeta} = c \cdot e^{j\delta}$$

$$c = |\underline{\zeta}| \quad \cos(\omega t + \theta) = \frac{1}{2} (e^{j(\omega t + \theta)} + e^{-j(\omega t + \theta)})$$

$$\delta = \arg(\underline{\zeta}) \quad \cos(\omega t + \theta) = \operatorname{Re}(e^{j(\omega t + \theta)})$$

$$U_m e^{-dt} \cos(\omega t + \theta) = U_m e^{-dt} \operatorname{Re}(e^{j(\omega t + \theta)}) = \operatorname{Re}(\underline{\zeta})$$

$$\operatorname{Re}(c \cdot e^{-dt}) = U_m c \cdot \operatorname{Re}(e^{j(\omega t + \theta)}) = c \cdot \operatorname{Re}(e^{j\delta})$$

$$c = U_m e^{-dt}$$

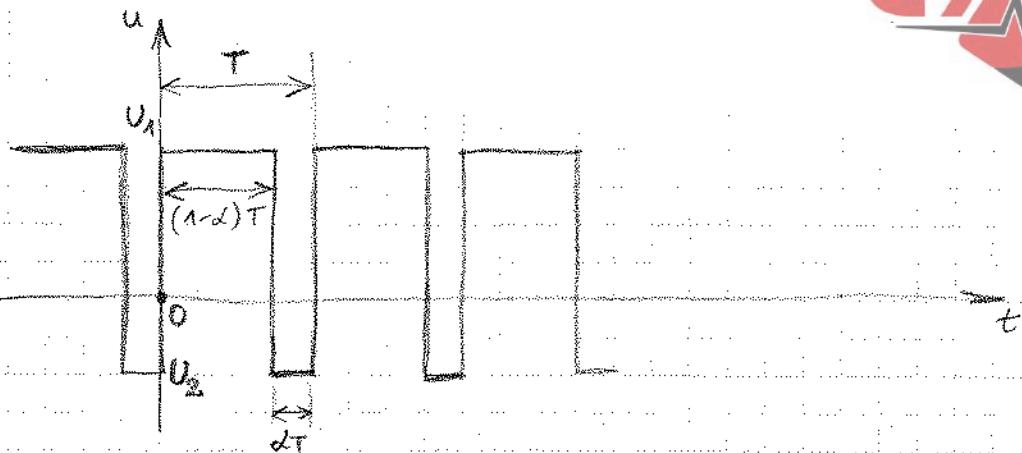
$$\delta = \omega t + \theta$$

$$\underline{\zeta} = U_m e^{-dt} e^{j(\omega t + \theta)}$$

$$\underline{\zeta} = U_m e^{-dt + j(\omega t + \theta)}$$

$$\underline{\zeta} = U_m e^{-dt + j(\omega t + \theta)}$$

22.



$0 \leq t \leq T \rightarrow \text{ЈЕДАН ПЕРИОД}$

$$u(t) = \begin{cases} U_1, & 0 < t < (1-\alpha)T \\ U_2, & (1-\alpha)T < t < T \end{cases}$$

* НАПОН ЈЕ ПРЕКИДНА ФУНКЦИЈА У ТРЕНУЦИМА:

$$t = kT \quad \wedge \quad t = kT + (1-\alpha)T, \quad k \in \mathbb{Z} \quad (k=0, \pm 1, \pm 2, \dots)$$

У ТРЕНУЦИМА ПРЕКИДА НАПОН НИЈЕ ДЕФИНИСАНО!

23. СРЕДЊА ВРЕДНОСТ НАПОНА:

$$U_{SR} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} u(t) dt$$

$0 < t < T \rightarrow \text{ЈЕДАН ПЕРИОД}$

$$U_{SR} = \frac{1}{T} \int_0^T u(t) dt$$

$$u(t) = \begin{cases} U_1, & 0 < t < (1-\alpha)T \\ U_2, & (1-\alpha)T < t < T \end{cases}$$

$$U_{SR} = \frac{1}{T} \left(\int_0^{(1-\alpha)T} U_1 dt + \int_{(1-\alpha)T}^T U_2 dt \right) = \frac{1}{T} (U_1(1-\alpha)T + U_2(\alpha T))$$

$$U_{SR} = \frac{1}{T} T (U_1(1-\alpha) + U_2\alpha) = \boxed{U_1(1-\alpha) + U_2\alpha}$$

$$\boxed{U_{SR} = (1-\alpha)U_1 + \alpha U_2}$$

24. ЕФЕКТИВНА ВРЕДНОСТ НАПОНА:

$$U_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T u(t)^2 dt}$$

$$U_{\text{eff}} = \sqrt{\frac{1}{T} \left(\int_0^{(1-\alpha)T} U_1^2 dt + \int_{(1-\alpha)T}^T U_2^2 dt \right)}$$

$$U_{\text{eff}} = \sqrt{\frac{1}{T} (U_1^2 (1-\alpha)T + U_2^2 (T - (1-\alpha)T))}$$

$$U_{\text{eff}} = \sqrt{\frac{1}{T} T (U_1^2 (1-\alpha) + U_2^2 \alpha)}$$

$$U_{\text{eff}} = \sqrt{(1-\alpha) U_1^2 + \alpha U_2^2}$$

ОДРЕЗИТИ

25. $i(t) = I_m \cos(\omega t + \psi)$

ЕФЕКТИВНА ВРЕДНОСТ СТРУЈЕ!

$$I_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T i(t)^2 dt}$$

$$\cos^2 \frac{\omega}{2} = \frac{1 + \cos \omega}{2}$$

$$I_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T I_m^2 \cos^2(\omega t + \psi) dt}$$

$$I_{\text{eff}} = \sqrt{\frac{1}{T} I_m^2 \int_0^T \frac{1}{2} (1 + \cos(2\omega t + 2\psi)) dt}$$

$$I_{\text{eff}} = \sqrt{\frac{1}{T} I_m^2 \frac{1}{2} \left(\int_0^T dt + \int_0^T \cos(2\omega t + 2\psi) dt \right)}$$

$$I_{\text{eff}} = \sqrt{\frac{I_m^2}{2T} (T + \sin(2 \cdot \frac{2\pi}{\omega} \cdot T + 2\psi) - \sin(2 \cdot \omega \cdot 0 + 2\psi))}$$

$$I_{\text{eff}} = \sqrt{\frac{I_m^2}{2T} (T + \sin(4\pi + 2\psi) - \sin 2\psi)}$$

$$I_{\text{eff}} = \sqrt{\frac{I_m^2}{2T} (T + \cancel{\sin 9\pi} \cdot \cos 2\psi + \cancel{\cos 9\pi} \cdot \sin 2\psi - \sin 2\psi)}$$

$$I_{\text{eff}} = \sqrt{\frac{I_m^2}{2T} (T + \cancel{\sin 2\psi} - \sin 2\psi)} = \sqrt{\frac{I_m^2}{2T} \cdot T}$$

$$I_{\text{eff}} = I_m \frac{1}{\sqrt{2}} = \frac{I_m \sqrt{2}}{2}$$

$$26. i(t) = I_0 + I_m \cos \omega t$$

$$i(t) = i(t+T)$$

$$I_0 + I_m \cos \omega t = I_0 + I_m \cos(\omega t + \omega T)$$

$$\omega T = 2\pi$$

$$T = \frac{2\pi}{\omega}$$

ОСНОВНОЙ
ПЕРИОД

$$I_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

$$I_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T (I_0 + I_m \cos \omega t)^2 dt}$$

$$I_{\text{eff}} = \sqrt{\frac{1}{T} \left(\int_0^T I_0^2 dt + \int_0^T I_m^2 \cos^2 \omega t dt + 2 \int_0^T I_0 I_m \cos \omega t dt \right)}$$

$$I_{\text{eff}} = \sqrt{\frac{1}{T} \left(I_0^2 T + \frac{1}{2} I_m^2 \left(\int_0^T dt + \int_0^T \cos 2\omega t dt \right) + 2 I_0 I_m \cdot (\sin 2 \cdot \frac{2\pi}{\omega} \cdot T - \sin 0) \right)}$$

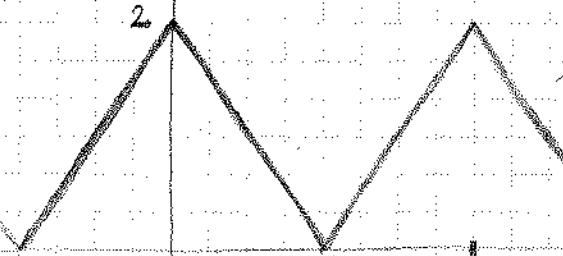
$$I_{\text{eff}} = \sqrt{\frac{1}{T} \left(I_0^2 T + \frac{I_m^2}{2} (T + \sin 2 \cdot \frac{2\pi}{\omega} \cdot T - \sin 0) + 2 I_0 I_m \cdot \sin 0 \right)}$$

$$I_{\text{eff}} = \sqrt{\frac{1}{T} I_0^2 T + \frac{I_m^2}{2} T + \frac{1}{2} \sin 0}$$

$$I_{\text{eff}} = \sqrt{I_0^2 + \frac{I_m^2}{2}}$$

израсчнать среднюю и эффективную вредность:

$i(t)$



$$i(t) = \begin{cases} -\frac{4}{T}t + 2 & 0 < t < \frac{T}{2} \\ \frac{4}{T}t - 2 & \frac{T}{2} < t < T \end{cases}$$

$$I_{\text{ср}} = \frac{1}{T} \int_0^T i(t) dt$$

$$y = kx + u$$

$$i = k \cdot t + u$$

$$1) t=0$$

$$i=u \Rightarrow u=2$$

$$2) t=T/2 \quad i=0$$

$$0 = k \cdot T/2 + u \quad \leftarrow 0$$

$$t=T \quad i=2$$

$$2 = k \cdot T + u \quad \leftarrow +$$

$$k = -\frac{4}{T}$$

$$I_{SR} = \frac{1}{T} \left(\int_0^{T/2} \left(-\frac{4}{T}t + 2 \right) dt + \int_{T/2}^T \left(\frac{4}{T}t - 2 \right) dt \right)$$

$$I_{SR} = \frac{1}{T} \left(-\frac{4}{T} \cdot \frac{T^2}{4} + 2 \cdot \frac{T}{2} + \frac{4}{T} \cdot \frac{T^2}{2} - \frac{4}{T} \cdot \frac{T}{2} \right) = 2(T - T/2)$$

$$I_{SR} = \frac{1}{T} \left(-\frac{2T^2}{2} + T + \frac{2}{T} \cdot \frac{3}{2} T^2 - 2 \cdot \frac{T}{2} \right) = \frac{1}{T} \left(-\frac{T}{2} + T + \frac{3}{2} T - T \right)$$

$$I_{SR} = \frac{1}{T} \frac{2}{2} T \Rightarrow I_{SR} = 1A$$

$$I_{eff} = \sqrt{\frac{1}{T} \left(\int_0^{T/2} \left(-\frac{4}{T}t + 2 \right)^2 dt + \int_{T/2}^T \left(\frac{4}{T}t - 2 \right)^2 dt \right)}$$

$$I_{eff} = \sqrt{\frac{1}{T} \left(\int_0^{T/2} \frac{16}{T^2} t^2 dt + \int_0^{T/2} \frac{16}{T} t dt + \int_0^{T/2} 4 dt + \int_{T/2}^T \frac{16}{T^2} t^2 dt - \int_{T/2}^T \frac{16}{T} t dt + \int_{T/2}^T 4 dt \right)}$$

$$I_{eff} = \sqrt{\frac{1}{T} \left(\frac{16}{T^2} \cdot \frac{1}{3} \left(\frac{T^2}{8} - 0 \right) - \frac{16}{T} \cdot \frac{1}{2} \left(\frac{T^2}{4} - 0 \right) + \frac{2}{T} + \frac{16}{T^2} \cdot \frac{1}{3} \left(T^3 - \frac{T^3}{8} \right) - \frac{16}{T} \cdot \frac{1}{2} \left(T^2 - \frac{T^2}{4} \right) + \frac{2}{T} \right)}$$

$$I_{eff} = \sqrt{\frac{1}{T} \left(\frac{2}{3} T - 2T + 2T + \frac{2}{T} \cdot \frac{1}{3} \cdot \frac{7}{8} T^2 - \frac{8}{T} \cdot \frac{3}{4} T^2 + 2T \right)}$$

$$I_{eff} = \sqrt{\frac{1}{T} \left(\frac{2}{3} T + \frac{19}{3} T - 6T + 2T \right)} = \sqrt{\frac{1}{T} \left(\frac{16}{3} - 4 \right)} = \sqrt{\frac{16 - 12}{3}}$$

$$I_{eff} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} \Rightarrow I_{eff} = \frac{2\sqrt{3}}{3}$$

28. израчунати средњу и ефективну вредност:

$$i(t) = I_{max} \cos^2 \left(\frac{2\pi t}{T} + \theta \right)$$

$$I_{max} = 10mA \quad T = 20ms$$

$$I_{SR} = \frac{1}{T} \int_0^T i(t) dt = \frac{1}{T} \int_0^T I_{max} \cos^2 \left(\frac{2\pi t}{T} + \theta \right) dt$$

$$I_{SR} = \frac{1}{T} I_{max} \cdot \int_0^T \frac{1}{2} (1 + \cos(\frac{4\pi t}{T} + 2\theta)) dt$$

$$I_{SR} = \frac{1}{T} I_{max} \cdot \frac{1}{2} \left(\int_0^T dt + \int_0^T \cos(\frac{4\pi t}{T} + 2\theta) dt \right)$$

$$I_{SR} = \frac{1}{2T} I_{max} \left(T + \sin \left(\frac{4\pi T}{T} + 2\theta \right) - \sin \left(\frac{4\pi \cdot 0}{T} + 2\theta \right) \right)$$

$$I_{SR} = \frac{I_{max}}{2T} \cdot T + \frac{I_{max}}{2T} \left(\sin 4\pi \cdot \cos 2\theta + \sin 2\theta \cdot \cos 4\pi - \sin 2\theta \right)$$

$$T = I_{max} = 10mA \Rightarrow I_{SR} = 5mA$$

$$I_{eff} = \sqrt{\frac{1}{T} \int_0^T i(t)^2 dt}$$

$$I_{eff} = \sqrt{\frac{1}{T} \int_0^T I_{max}^2 \cdot \cos^2\left(\frac{2\pi t}{T} + \theta\right) dt}$$

$$I_{eff} = \sqrt{\frac{1}{T} I_{max}^2 \int_0^T \frac{1}{4}(1 + \cos(\frac{2\pi t}{T} + \theta))^2 dt}$$

$$I_{eff} = \sqrt{\frac{I_{max}^2}{T} \cdot \frac{1}{4} \left(\int_0^T dt + \int_0^T 2 \cos\left(\frac{2\pi t}{T} + \theta\right) dt + \int_0^T \cos^2\left(\frac{2\pi t}{T} + \theta\right) dt \right)}$$

$$I_{eff} = \sqrt{\frac{I_{max}^2}{4T} (T + 2(\sin(2\pi + \theta) - \sin \theta) + \frac{1}{2} \left(\int_0^T dt + \int_0^T \cos\left(\frac{2\pi t}{T} + \theta\right) dt \right))}$$

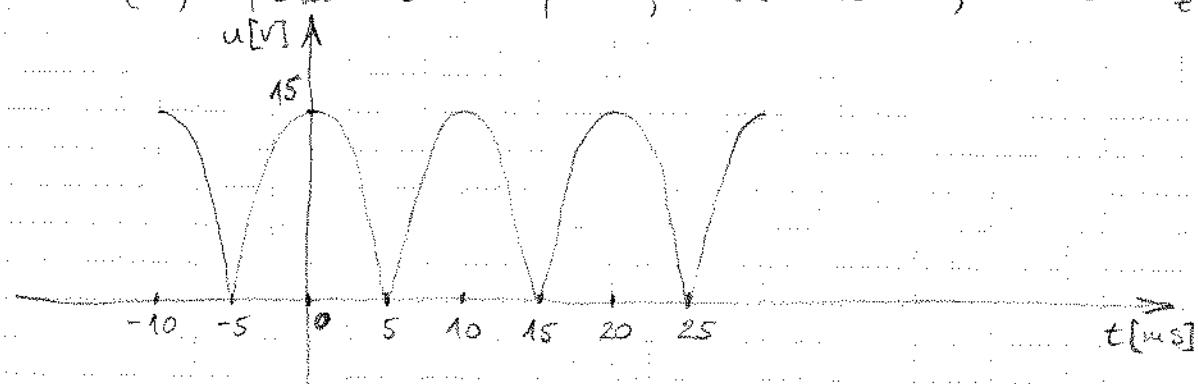
$$I_{eff} = \sqrt{\frac{I_{max}^2}{4T} (T + 2(\cancel{\sin 2\pi} \cos^2 \theta + \cancel{\sin \theta} \cos 2\pi - \sin \theta) + \frac{1}{2} (T + \sin(2\pi + \theta) - \sin \theta))}$$

$$I_{eff} = \sqrt{\frac{I_{max}^2}{4T} (T + \frac{1}{2}T)} = \sqrt{\frac{I_{max}^2}{4T} \frac{3}{2} T} = \frac{I_{max}}{2} \sqrt{\frac{3}{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$I_{eff} = \frac{I_{max}}{4} \sqrt{6} = \frac{10}{4} \sqrt{6} \text{ mA} \Rightarrow \boxed{I_{eff} = \frac{5}{2} \sqrt{6} \text{ mA}}$$

29. ИЗРАЧУНАТИ ПЕРІОД, ЕФЕКТИВНУ ВРЕДНОСТЬ І СРЕДНЮ ВРЕДНОСТЬ

$$u(t) = |U_m \cos 2\pi f t|, U_m = 15V, f = 50Hz$$



$$\therefore u(t) = u(t+T)$$

$$|U_m \cos 2\pi f t| = |U_m \cos (2\pi f t + 2\pi f T)|$$

$$2\pi f T = k\pi$$

$$\boxed{T = \frac{1}{2f}} \Rightarrow T = \frac{1}{2 \cdot 50 \text{ s}^{-1}} = 0.01 \text{ s} = 10 \text{ ms}$$

$$U_{SR} = \frac{1}{T} \int_0^T u(t) dt = \frac{1}{T} \int_0^T |U_m \cos 2\pi f t| dt$$

$$U_{SR} = \frac{1}{T} \int_{-T/2}^{T/2} U_m \cos 2\pi f t dt = \frac{1}{T} U_m \left(\sin \left(2\pi f \frac{T}{2} \right) - \sin \left(2\pi f \left(-\frac{T}{2} \right) \right) \right)$$

$$U_{SR} = \frac{U_m}{T} \left(2 \sin \left(\pi f T \right) \right) = \frac{U_m}{T} \cdot 2 \sin \left(\pi \frac{1}{f} \right)$$

$$U_{eff} = \sqrt{\frac{1}{T} \int_0^T |U_m \cos 2\pi f t|^2 dt} = \sqrt{\frac{1}{T} \int_0^T U_m^2 \cos^2(2\pi f t) dt}$$

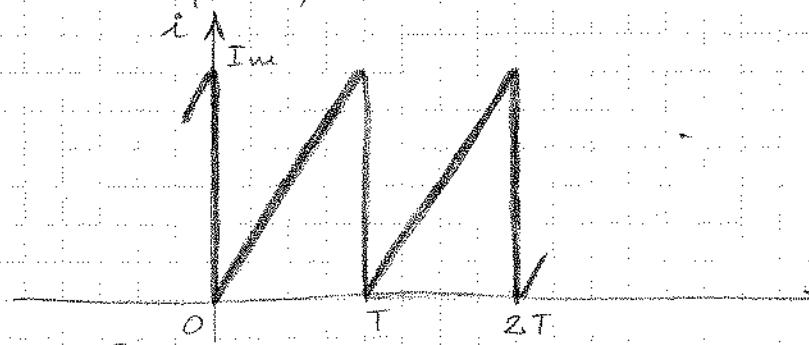
$$U_{eff} = \sqrt{\frac{1}{T} U_m^2 \int_0^T \cos^2(2\pi f t) dt} = \sqrt{\frac{1}{T} U_m^2 \frac{1}{2} \int_0^T (1 + \cos 4\pi f t) dt}$$

$$U_{eff} = \sqrt{\frac{1}{T} U_m^2 \frac{1}{2} \left(T + \frac{1}{4\pi f} (\sin 4\pi f - \sin 0) \right)} = \sqrt{\frac{1}{T} U_m^2 \frac{1}{2} \cdot T} = \sqrt{\frac{U_m^2}{2}}$$

$$U_{eff} = \frac{U_m}{\sqrt{2}} = \frac{15}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{15\sqrt{2}}{2} V \Rightarrow U_{eff} \approx 10.61 V$$

30. Определите среднюю и действующую мощность:

$$i(t) = \frac{I_m}{T} t, \quad 0 < t < T$$



$$I_{SR} = \frac{1}{T} \int_0^T i(t) dt = \frac{1}{T} \int_0^T \frac{I_m}{T} t dt = \frac{1}{T} \frac{I_m}{T} \cdot \frac{t^2}{2} \Big|_0^T = \frac{I_m}{T^2} \frac{1}{2} (T^2 - 0)$$

$$I_{SR} = \frac{I_m}{2} \frac{1}{T^2} T^2 \Rightarrow I_{SR} = \frac{I_m}{2}$$

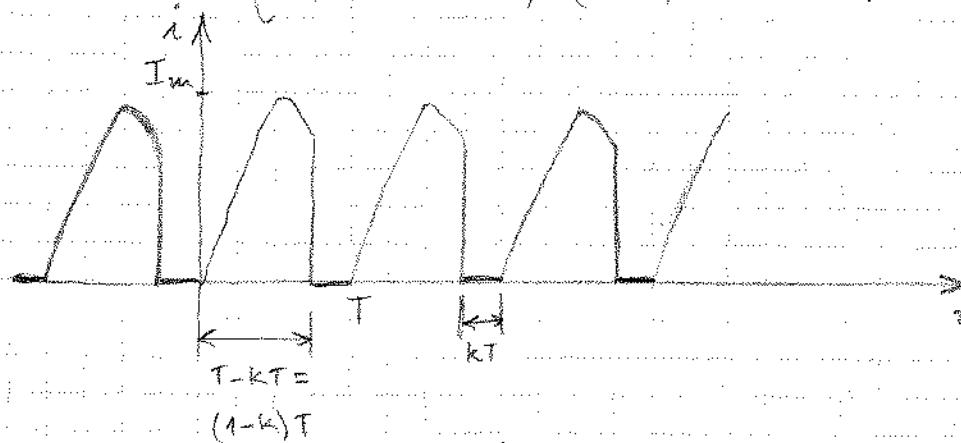
$$I_{eff} = \sqrt{\frac{1}{T} \int_0^T \frac{I_m^2}{T^2} t^2 dt} = \sqrt{\frac{1}{T} \frac{I_m^2}{T^2} \frac{t^3}{3} \Big|_0^T} = \sqrt{\frac{I_m^2}{3} \frac{1}{T^3} (T^3 - 0)}$$

$$I_{eff} = \sqrt{\frac{I_m^2}{3} \frac{1}{T^2}} = |I_m \sqrt{3}|$$

$$0 < k < 1$$

$$0 \leq t \leq T$$

* 31. $i(t) = \begin{cases} I_m \sin \frac{\pi t}{T}, & 0 \leq t < (1-k)T \\ 0, & (1-k)T \leq t \leq T \end{cases}$



$$I_{SR} = \frac{1}{T} \int_0^T i(t) dt = \frac{1}{T} \int_0^{(1-k)T} I_m \sin \frac{\pi t}{T} dt = \frac{I_m}{\pi} \left[-\cos \frac{\pi t}{T} \right]_0^{(1-k)T} = \frac{I_m}{\pi} \left(-\cos \pi (1-k) + \cos 0 \right)$$

$$I_{SR} = \frac{I_m}{\pi} \left(-\cos (\pi - k\pi) + 1 \right) = \frac{I_m}{\pi} \left(-\left(\frac{\cos \pi}{-1} \cdot \cos k\pi - \frac{\sin \pi}{-1} \cdot \sin k\pi \right) + 1 \right)$$

$$\boxed{I_{SR} = \frac{I_m}{\pi} (1 + \cos k\pi)}$$

$$I_{eff} = \sqrt{\frac{1}{T} \int_0^{(1-k)T} I_m^2 \sin^2 \frac{\pi t}{T} dt} = \sqrt{\frac{1}{T} I_m^2 \frac{1}{2} \int_0^{(1-k)T} (1 - \cos \frac{2\pi t}{T}) dt}$$

$$I_{eff} = \sqrt{\frac{I_m^2}{2T} \left((1-k)T - \frac{T}{2\pi} \left(\sin \frac{2\pi}{T} (1-k)\pi - \sin 0 \right) \right)}$$

$$I_{eff} = \sqrt{\frac{I_m^2}{2T} \left((1-k)T - \frac{T}{2\pi} \left(\sin \frac{2\pi}{T} (1-k)\pi - \sin k\pi \cdot \frac{\cos \pi}{-1} - \frac{\sin \pi}{-1} \right) \right)}$$

$$I_{eff} = \sqrt{\frac{I_m^2}{2T} (1-k)T + \frac{I_m^2}{2T} \frac{T}{2\pi} (-\sin k\pi)}$$

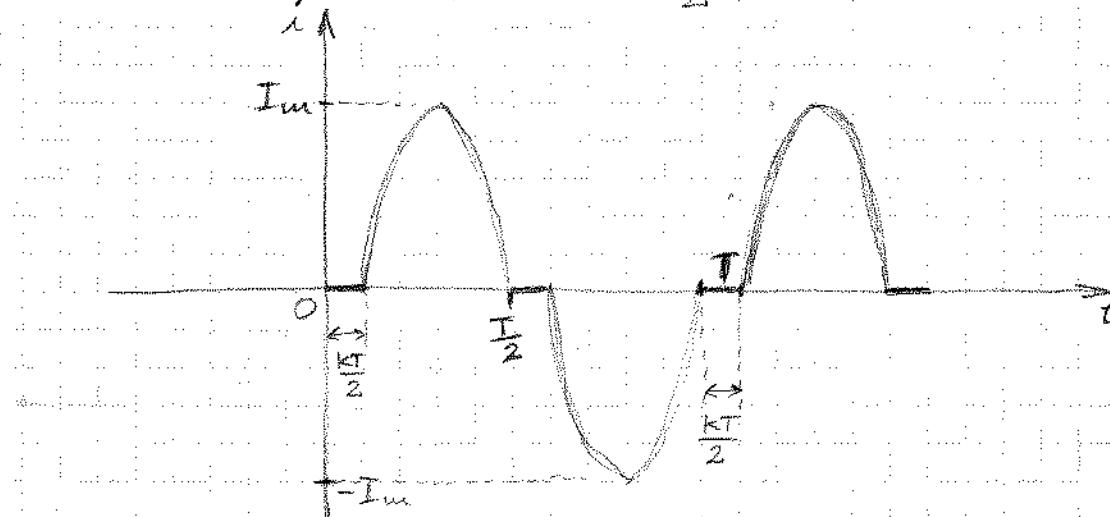
$$\boxed{I_{eff} = \sqrt{I_m^2 \left(\frac{1-k}{2} + \frac{\sin 2k\pi}{4\pi} \right)}}$$

$$I_{eff} = I_m \sqrt{\frac{1-k}{2} + \frac{\sin 2k\pi}{4\pi}}$$

*32.

$$0 \leq t \leq T$$

$$i(t) = \begin{cases} 0 & , 0 \leq t < \frac{kT}{2} \\ I_m \sin \frac{2\pi t}{T}, & \frac{kT}{2} \leq t \leq \frac{T}{2} \\ 0 & , \frac{T}{2} \leq t < \frac{T(1+k)}{2} \\ I_m \sin \frac{2\pi t}{T}, & \frac{T(1+k)}{2} \leq t \leq T \end{cases} \quad 0 \leq k \leq 1$$



$$I_{eff} = \sqrt{\frac{1}{T} \left(\int_{\frac{kT}{2}}^{\frac{T}{2}} I_m^2 \sin^2 \frac{2\pi t}{T} dt + \int_{\frac{T}{2}}^{\frac{T(1+k)}{2}} I_m^2 \sin^2 \frac{2\pi t}{T} dt \right)}$$

$$I_{eff} = \sqrt{\frac{1}{T} I_m^2 \left(\int_{\frac{kT}{2}}^{\frac{T}{2}} dt - \int_{\frac{kT}{2}}^{\frac{T}{2}} \cos \frac{4\pi t}{T} dt + \int_{\frac{T}{2}}^{\frac{T(1+k)}{2}} dt - \int_{\frac{T}{2}}^{\frac{T(1+k)}{2}} \cos \frac{4\pi t}{T} dt \right)}$$

$$I_{eff} = \sqrt{\frac{1}{2T} I_m^2 \left((1-k)\frac{T}{2} - \frac{T}{4\pi} (\sin 2k\pi - \sin 2\pi) + \left(1 - \frac{1+k}{2}\right)T - \frac{T}{4\pi} (\sin 4\pi - \sin 2\pi(1+k)) \right)}$$

$$I_{eff} = \sqrt{\frac{1}{2T} I_m^2 \left(\frac{1-k}{2}T + \frac{T}{4\pi} \sin 2k\pi + \frac{1-k}{2}T + \frac{T}{4\pi} \sin 2\pi(1+k) \right)}$$

$$I_{eff} = \sqrt{\frac{I_m^2}{2T} ((1-k)T + \frac{T}{4\pi} (\sin 2k\pi + \sin (1+k)2\pi))}$$

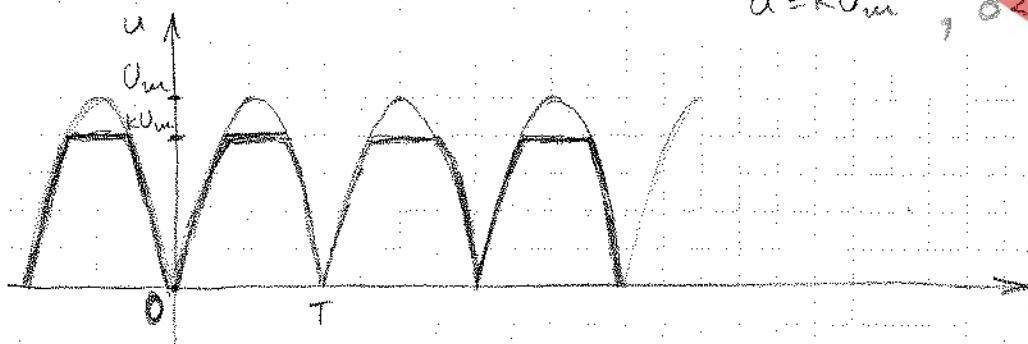
$$I_{eff} = \sqrt{\frac{I_m^2}{2T} (1-k)T + \frac{I_m^2}{2T} \frac{T}{4\pi} (\sin 2k\pi + \sin 2\pi \cos 2k\pi + \sin 2\pi \cos 2\pi)}$$

$$I_{eff} = \sqrt{I_m^2 \left(\frac{1-k}{2} + \frac{2 \sin 2k\pi}{2 \cdot 4\pi} \right)}$$

$$I_{eff} = I_m \sqrt{\frac{1-k}{2} + \frac{\sin 2k\pi}{4\pi}}$$

$$u = kU_m, \quad 0 < k < 1$$

* 33.



$$u(t) = \begin{cases} U_m \sin \frac{\pi t}{T}, & 0 \leq t \leq t_1 \quad \text{and} \quad (T-t_1) \leq t \leq T \\ kU_m, & t_1 < t < (T-t_1) \end{cases}, \quad t_1 = \frac{T}{\pi} \arcsin k$$

$$U_{SR} = \frac{1}{T} \left(\int_{t_1}^{t_1} U_m \sin \frac{\pi t}{T} dt + \int_{t_1}^{T-t_1} kU_m dt + \int_{T-t_1}^T U_m \sin \frac{\pi t}{T} dt \right)$$

$$U_{SR} = \frac{1}{T} \left(U_m \frac{T}{\pi} \left(-\cos \frac{T}{\pi} \arcsin k + \cos 0 \right) + U_m k (T - 2 \cdot \frac{T}{\pi} \arcsin k) \right)$$

$$U_{SR} = \frac{1}{T} \left(\frac{U_m T}{\pi} \left(1 - \cos \left(\frac{T}{\pi} \arcsin k \right) \right) + U_m k T - 2 \frac{U_m k T}{\pi} \arcsin k \right. \\ \left. + \frac{U_m T}{\pi} \left(\cos \left(\pi - \frac{T}{\pi} \arcsin k \right) + 1 \right) \right)$$

$$U_{SR} =$$

34.

35.



36. ОСНОВНИ ПЕРИОД: $u(t) \rightarrow T$

СРЕДЊА ВРЕДНОСТ: $u(t) \rightarrow U_{SR}$

ЕФЕКТИВНА ВРЕДНОСТ: $u(t) \rightarrow U$

$$U_2(t) = u(t) + U_0, \quad U_0 = \text{const}$$

• ОСНОВНИ ПЕРИОД $U_2(t) \rightarrow T$

$$U_{2SR} = U_{SR} + U_0$$

$$U_{2eff} = \sqrt{\frac{1}{T} \int_0^T (u(t) + U_0)^2 dt} = \sqrt{\frac{1}{T} \left(\int_0^T u^2(t) dt + \int_0^T 2u(t) \cdot U_0 dt + \int_0^T U_0^2 dt \right)}$$

$$U_{2eff} = \sqrt{\frac{1}{T} (U^2 T + 2U_0 \cdot U_{SR} T + U_0^2 T)}$$

$$U_{2eff} = \sqrt{U^2 + 2U_0 U_{SR} + U_0^2}$$

$$37. \quad u(t) = (5 + 10 \sin \omega t - 10 \cos \omega t) V$$

$$u(t) = 10(\sin \omega t - \cos \omega t) + 5$$

$$u(t) = 10 \left(\cos \left(\omega t + \frac{\pi}{2} \right) - \cos \omega t \right) + 5$$

$$u(t) = 5 - 10 \cdot (\cos \omega t - \cos \left(\omega t + \frac{\pi}{2} \right))$$

$$u(t) = 5 - 10 \cdot 2 \cdot \sin \frac{\omega t + \omega t + \frac{\pi}{2}}{2} \cdot \sin \frac{\omega t - \omega t - \frac{\pi}{2}}{2}$$

$$u(t) = 5 - 10 \cdot 2 \cdot \sin \left(\omega t - \frac{\pi}{4} \right) \cdot \frac{\sqrt{2}}{2}$$

$$u(t) = 5 - 10\sqrt{2} \sin \left(\omega t - \frac{\pi}{4} \right)$$

$$u(t) = 5 - 10\sqrt{2} \cos \left(\omega t + \frac{\pi}{4} \right)$$

$$U_{1eff} = \sqrt{\frac{1}{T} 200 \int_0^T \cos^2 \left(\omega t + \frac{\pi}{4} \right) dt} = \sqrt{\frac{200}{T} \left(T + \frac{1}{2\omega} \left(\sin(2\omega T + \frac{\pi}{4}) - \sin \frac{\pi}{4} \right) \right)}$$

$$U_{1eff} = \sqrt{\frac{100}{T} \left(T + \frac{1}{4\omega} \left(\sin 4\omega T \cdot \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos 4\omega T - \sin \frac{\pi}{4} \right) \right)}$$

$$U_{1eff} = 100, \quad (U_{SR} = 0)$$

$$U_{eff} = \sqrt{100 + 2 \cdot 5 \cdot 0 + 25} = \sqrt{125} = 5\sqrt{5} V \approx 11,2 V$$

41. $I_{\text{eff}} = I = 4A$

$$f = 100 \text{ Hz}$$

$$t_1 = 5 \text{ ms} \xrightarrow{\text{УСТАНОВКА}} i_1 = 2\sqrt{2} A \text{, т.е. синус}$$

опыту, израс: $i(t) = I_m \cos(\omega t + \psi)$

$$I_m = I\sqrt{2} = 4\sqrt{2} A$$

$$\omega = 2\pi f = 2\pi \cdot 100 \text{ s}^{-1} = 200\pi \text{ s}^{-1}$$

$$i_1(t_1) = I_m \cos(\omega t_1 + \psi)$$

$$\Rightarrow \frac{i_1}{I_m} = \cos(\omega t_1 + \psi)$$

$$\omega t_1 + \psi = \arccos\left(\frac{i_1}{I_m}\right) = \arccos\left(\frac{12\sqrt{2}}{2\sqrt{2}\sqrt{2}}\right) = \arccos\frac{1}{2}$$

$$\omega t_1 + \psi = \frac{\pi}{3}$$

$$\omega t_1 + \psi = -\frac{\pi}{3}$$

$$\omega t_1 = 200\pi \cdot \frac{1}{2} \cdot \frac{1}{100} = \pi$$

СТРУЖА $i(t)$ ОНАДА \times ТРЕНУТКУ t_1

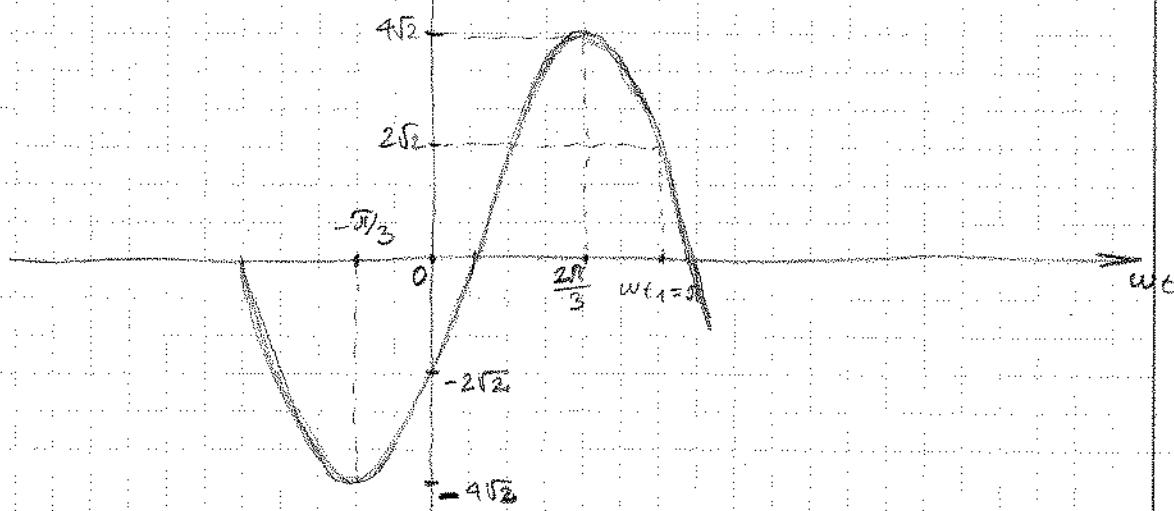
$$\Rightarrow \omega t_1 + \psi = \frac{\pi}{3}$$

$$\psi = \frac{\pi}{3} - \omega t_1 = \frac{\pi}{3} - \pi = -\frac{2\pi}{3}$$

$$\psi = -\frac{2\pi}{3}$$

$$\Rightarrow i(t) = 4\sqrt{2} \cos\left(200\pi t - \frac{2\pi}{3}\right)$$

$i(t) [A]$



42. $I_{\text{eff}} = I = 50 \mu\text{A}$

$$t_1 = 50 \mu\text{s} \rightarrow i(t_1) = I \text{ и опада}$$

$$\Delta t = 100 \mu\text{s} \rightarrow \text{изменя} \Delta \text{t} \text{ суседне} \text{ импульсы}$$

$$i(t) = I_m \cos(\omega t + \varphi)$$

$$I_m = I\sqrt{2} = 50\sqrt{2} \mu\text{A}$$

$$i(t) = I\sqrt{2} \cos\left(\frac{2\pi}{T}(t + \tau)\right), \tau \rightarrow \begin{array}{l} \text{помечено} \\ \text{импульс} \end{array} \begin{array}{l} \text{имеет} \\ \text{стабильную} \end{array} \begin{array}{l} \text{частоту} \\ \text{струю} \end{array}$$

$$T = 2\Delta t = 200 \mu\text{s}$$

$$i(t_1) = I\sqrt{2} \cos\left(\frac{2\pi}{T}(t_1 + \tau)\right) = I$$

$$\Rightarrow \cos\left(\frac{2\pi}{T}(t_1 + \tau)\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\frac{2\pi}{T}(t_1 + \tau) = \frac{\pi}{4}, \sqrt{\frac{2\pi}{T}(t_1 + \tau)} = \pm \frac{\pi}{4}$$

$$t_1 = 50 \mu\text{s} = \frac{T}{4}$$

КАКО У ТРЕХУТКУ t_1 ОПАДА

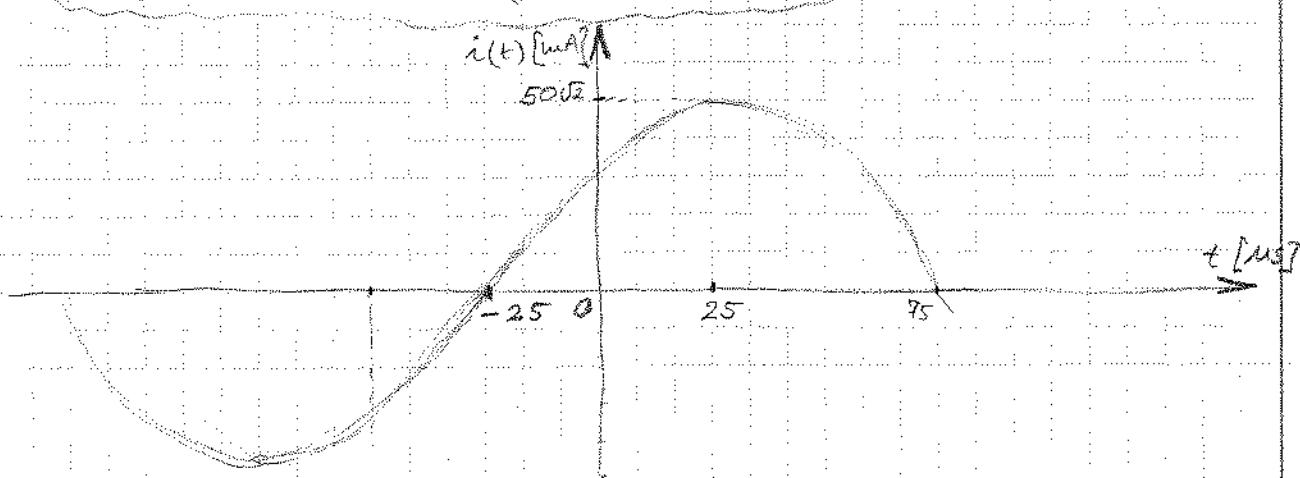
$$\Rightarrow \frac{2\pi}{T}(t_1 + \tau) = \frac{\pi}{4}$$

$$\frac{2\pi}{T} \cdot \frac{T}{4} + \frac{2\pi}{T} \tau = \frac{\pi}{4} \Rightarrow \frac{2\pi}{T} \tau = -\frac{\pi}{4}$$

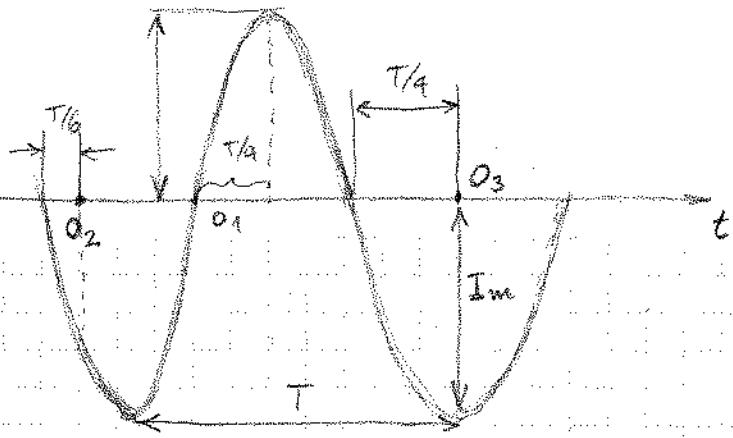
$$\Rightarrow \boxed{\tau = -\frac{T}{8}} \quad \boxed{\tau = -25 \mu\text{s}}$$

$$i(t) = I\sqrt{2} \cos\left(\frac{2\pi}{T}\left(t - \frac{T}{8}\right)\right)$$

$$\{ i(t) = 50\sqrt{2} \cos \pi \left(\frac{t}{100 \mu\text{s}} - \frac{1}{4} \right) \mu\text{A}$$



43.

a) ПОЧЕТНЫЙ ТРЕНУТАК У ТАЧКИ O_1 :

$$\tau = -\frac{T}{4} \text{ ПОЧЕТНЫЙ ТРЕНУТАК СТАБ}$$

$$i(t) = I_m \cos \left(\frac{2\pi}{T} (t + \tau) \right)$$

$$i(t) = I_m \cos \left(\frac{2\pi}{T} \left(t - \frac{T}{4} \right) \right)$$

$$i(t) = I_m \cos \left(wt - \frac{\pi}{2} \right)$$

d) ПОЧЕТНЫЙ ТРЕНУТАК У ТАЧКИ O_2 :

$$\tau = -\left(\frac{T}{4} + \frac{T}{4} + \left(\frac{T}{4} - \frac{T}{6}\right)\right) = -\left(\frac{T}{2} + \frac{3-2}{12}T\right) = -\frac{6+1}{12}T$$

$$\tau = -\frac{7}{12}T \text{ ПОЧЕТНЫЙ ТРЕНУТАК СТАБ}$$

$$i(t) = I_m \cos \left(\frac{2\pi}{T} (t + \tau) \right) = I_m \cos \left(\frac{2\pi}{T} \left(t - \frac{7}{12}T \right) \right)$$

$$i(t) = I_m \cos \left(wt - \frac{7}{6}\pi \right)$$

$$i(t) = I_m \cos \left(wt + \frac{5}{6}\pi \right)$$

e) ПОЧЕТНЫЙ ТРЕНУТАК У ТАЧКИ O_3 :

$$\tau = (2 \cdot \frac{T}{4}) = \frac{T}{2} \text{ ПОЧЕТНЫЙ ТРЕНУТАК СТАБ}$$

$$i(t) = I_m \cos \left(\frac{2\pi}{T} (t + \tau) \right) = I_m \cos \left(\frac{2\pi}{T} \left(t + \frac{T}{2} \right) \right)$$

$$i(t) = I_m \cos \left(wt + \pi \right)$$

44. $i(t) = I\sqrt{2} \cos(\omega t + \Psi)$

ТРЕНУЧИ У КОЈИМА ОВА СТРУЈА МЕЊА СМЕР:

$$\omega t_k + \Psi = \frac{\pi}{2} + k\pi, \quad k=0, \pm 1, \pm 2, \pm 3, \dots$$

$$\omega t_k = \left(\frac{\pi}{2} + k\pi\right) - \Psi = \left(k + \frac{1}{2}\right)\pi - \Psi$$

$$t_k = \frac{\left(k + \frac{1}{2}\right)\pi - \Psi}{\omega}$$

45. $i_1(t) = 20 \cos\left(\omega t - \frac{5\pi}{12}\right) \text{mA}$

$$i_2(t) = 25 \sin\left(\omega t - \frac{5\pi}{6}\right) \text{mA}$$

a) упоредити по амплитуди

b) упоредити по фази

a) по амплитуди

$$\frac{I_{1m}}{I_{2m}} = \frac{20 \text{mA}}{25 \text{mA}} = 0.8 < 1$$

$$|i_1| < |i_2|$$

i_1 слабија од i_2 .

b) $i_1(t) = 20 \cos\left(\omega t - \frac{5\pi}{12}\right) \text{mA}$

$$i_2(t) = 25 \cos\left(\omega t - \frac{5\pi}{6} - \frac{\pi}{2}\right) = 25 \cos\left(\omega t - \frac{5+3}{6}\pi\right)$$

$$i_2(t) = 25 \cos\left(\omega t - \frac{8}{6}\pi\right) = 25 \cos\left(\omega t - \frac{4\pi}{3}\right)$$

$$i_2(t) = 25 \cos\left(\omega t + \frac{2\pi}{3}\right) \text{mA}$$

ФАЗНА РАЗЛИКА СТРУЈА: $\Delta = (\omega t + \Psi_1) - (\omega t + \Psi_2)$

$$\Delta = \Psi_1 - \Psi_2 = -\frac{5\pi}{12} - \frac{2\pi}{3} = -\frac{5+8}{12}\pi = -\frac{13}{12}\pi < -\pi$$

$$\Delta = -\frac{13}{12}\pi + 2\pi \Rightarrow \Delta = \frac{11}{12}\pi$$

\Rightarrow СТРУЈА i_1 ФАЗНО ПРЕДЊАЧИ СТРУЈУ i_2

3A

$$\frac{11\pi}{12}$$

46. $e_1 = 220 \cos(\omega t + \frac{\pi}{6}) V$

$$e_2 = ?$$

a) $\theta_1 - \theta_2 = \frac{2\pi}{3} = B_1$

$$\theta_2 = \theta_1 - \frac{2\pi}{3} = \frac{\pi}{6} - \frac{2\pi}{3} = \frac{1-4}{6}\pi = -\frac{3}{6}\pi$$

$$\boxed{\theta_2 = -\frac{\pi}{2}}$$

$$\boxed{e_2 = 220 \cos(\omega t - \frac{\pi}{2}) V}$$

b) $\theta_1 - \theta_2 = -\frac{\pi}{2} = B_2$

$$\theta_2 = \theta_1 + \frac{\pi}{2} = \frac{\pi}{6} + \frac{\pi}{2} = \frac{\pi + 3\pi}{6} = +\frac{4}{6}\pi = +\frac{2}{3}\pi$$

$$\boxed{\theta_2 = +\frac{2\pi}{3}}$$

$$\boxed{e_2 = 220 \cos(\omega t + \frac{2\pi}{3}) V}$$

c) $\theta_1 - \theta_2 = \pi$

$$\theta_2 = \frac{\pi}{6} - \pi = -\frac{5\pi}{6}$$

$$\boxed{e_2 = 220 \cos(\omega t - \frac{5\pi}{6}) V}$$

47. $f = 100 Hz = f_1 = f_2 = f_3 \Rightarrow \omega = 2\pi \cdot 100 s^{-1} = 200\pi s^{-1}$

$$I_{2m} = 4A$$

$$T = \frac{1}{100} s = 10 \mu s$$

$$I_{3m} = 8A$$

$$I_3 = 5I_1 \rightarrow I_3 = \frac{I_{2m}}{\sqrt{2}} + 5I_1 \Rightarrow I_1 = \frac{8\sqrt{2}}{105}$$

$$I_1 = \frac{4\sqrt{2}}{5} A \Rightarrow I_{1m} = \frac{8}{5} A = 1.6A$$

$$t_1 = 2 \mu s \rightarrow i_3(t) \text{ PACTE A MEIA SHAK}$$

$$\Psi_2 - \Psi_3 = \frac{2\pi}{3}$$

$$T_2 = -(2 \mu s + \frac{T}{2}) = -2 \mu s - \frac{10}{2} \mu s$$

$$\Psi_1 - \Psi_2 = -\frac{\pi}{2}$$

$$\boxed{T_3 = -7 \mu s} \quad \boxed{-\frac{7T}{10}}$$

$$i_3(t) = 8 \cos\left(\frac{2\pi}{T}(t - \frac{7T}{10})\right)$$

$$i_3(t) = 8 \cos(\omega t - \frac{7\pi}{5})$$

$$i_3(t) = 8 \cos(\omega t + \frac{3\pi}{5})$$

$$i_1(t) = 1.6 \cos(\omega t + \Psi_1) = 1.6 \cos(200\pi t + \Psi_1)$$

$$i_2(t) = 4 \cos(\omega t + \Psi_2) = 4 \cos(200\pi t + \Psi_2)$$

$$i_3(t) = 8 \cos(\omega t + \Psi_3) = 8 \cos(200\pi t + \Psi_3)$$

48. $i_1 = 3 \cos \omega t \text{ A}$

$$\left. \begin{array}{l} i_2 = 4 \cos(\omega t + \frac{\pi}{2}) \text{ A} \\ i_3 = 8 \cos(\omega t - \pi) \text{ A} \end{array} \right\} i = i_2 - i_1$$

$$i = I_m \cos(\omega t + \Psi)$$

$$I_m \cos(\omega t + \Psi) = 4 \cos(\omega t + \frac{\pi}{2}) \text{ A} - 3 \cos \omega t \text{ A}$$

$$I_m \cos(\omega t + \Psi) = (4 \cos(\omega t + \frac{\pi}{2}) + 3 \cos(\omega t - \pi)) \text{ A}$$

$$I_m = \sqrt{I_{1m}^2 + I_{2m}^2 + 2 I_{1m} I_{2m} \cos(\Psi_1 - \Psi_2)}$$

$$I_m = \sqrt{4^2 + 3^2 + 2 \cdot 4 \cdot 3 \cdot \cos(\frac{\pi}{2} + \pi)} = \sqrt{16 + 9} = \sqrt{25} = 5 \text{ A}$$

$$\Psi = -\arctg \frac{I_{1m}}{I_{2m}} = -\arctg \frac{4}{3} = -53,13^\circ$$

$$\Psi \approx 126.9^\circ$$

$$\boxed{i(t) = 5 \cos(\omega t + 126.9^\circ)}$$

49. $U_1 = 5 \sin \omega t \text{ V}$

$$U_2 = 10 \sin (\omega t + \frac{\pi}{2}) \text{ V}$$

ЕФЕКТИВНА ВРЕДНОСТ И ПОЧЕТНА ФАЗА:

a) $U = U_2 - U_1$

d) $U = U_1 - U_2$

b) $U = U_1 + U_2$

a) $U = U_m \cos (\omega t + \theta) = 10 \sin (\omega t + \frac{\pi}{2}) - 5 \sin \omega t$

$$U = 10 \cos \omega t - 5 \cos (\omega t - \frac{\pi}{2})$$

$$U = 10 \cos \omega t + 5 \cos (\omega t + \frac{\pi}{2})$$

$$U_m = \sqrt{U_{1m}^2 + U_{2m}^2 + 2 U_{1m} U_{2m} \cos (0 - \frac{\pi}{2})} = \sqrt{10^2 + 5^2}$$

$$U_m = \sqrt{125} \text{ V} \approx 11.2 \text{ V} \Rightarrow U = \frac{U_m}{\sqrt{2}} = \frac{\sqrt{125}}{\sqrt{2}} \approx 7.91 \text{ V}$$

$$\theta = -\arctg \frac{U_m}{U_{2m}} = -\arctg \frac{10}{5} = -\arctg 2$$

$$U = 11.2 \cos (\omega t)$$

d) $U = 5 \cos (\omega t - \frac{\pi}{2}) - 10 \cos \omega t$

$$U = 5 \cos (\omega t - \frac{\pi}{2}) + 10 \cos (\omega t + \pi)$$

$$\boxed{U \approx 7.91 \text{ V}}$$

$$\theta =$$

$$U = 11.2 \cos (\omega t)$$

b) $U = 5 \sin \omega t + 10 \sin (\omega t + \frac{\pi}{2})$

$$U = 5 \cos (\omega t - \frac{\pi}{2}) + 10 \cos \omega t$$

50. $i(t) = I_m \cos(\omega t + \Psi)$

$$I_m = 10A$$

$$f = 50 \text{ Hz}$$

a) БРЗИНА КОЈОМ СЕ СТРУЈА МЕВА

$$\frac{di}{dt} = -\omega I_m \sin(\omega t + \Psi) = \omega I_m \cos(\omega t + \Psi + \frac{\pi}{2})$$

АМПЛИТУДА БРЗИНЕ: $\left| \frac{di}{dt} \right|_{\max} = \omega I_m = 2\pi f I_m = 10000\pi \frac{A}{s}$

КАДА JE $\cos(\omega t_1 + \Psi + \frac{\pi}{2}) = 1 \Leftrightarrow \omega t_1 + \Psi + \frac{\pi}{2} = 2k\pi, k=0, \pm 1, \pm 2, \dots$

$$\underline{\omega t_1 = 2k\pi - \frac{\pi}{3}} = 2k\pi - \frac{\pi}{2} - \frac{\pi}{6} \quad \text{БРЗИНА ДОСТИЖЕ MAX, СТРУЈА} \rightarrow \text{РАСТЕ!}$$

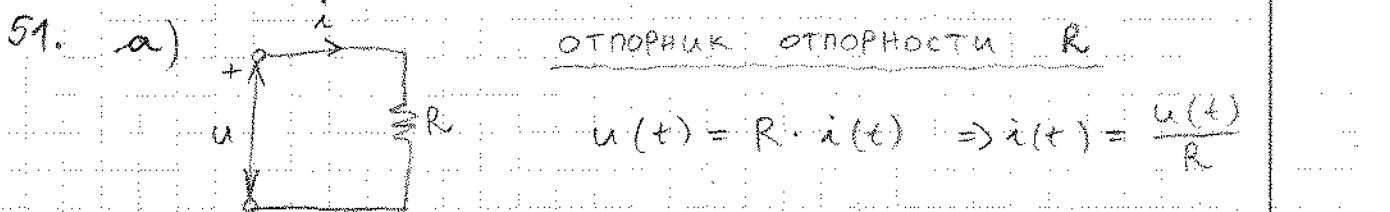
КАДА JE $\cos(\omega t_2 + \Psi + \frac{\pi}{2}) = -1 \Leftrightarrow \omega t_2 + \Psi + \frac{\pi}{2} = (2k+1)\pi$

$$\underline{\omega t_2 = 2k\pi + \pi - \frac{\pi}{2} - \frac{\pi}{6}} = 2k\pi + \frac{\pi}{3} \quad \text{БРЗИНА ДОСТИЖЕ MIN, СТРУЈА} \rightarrow \text{ОПАДА!}$$

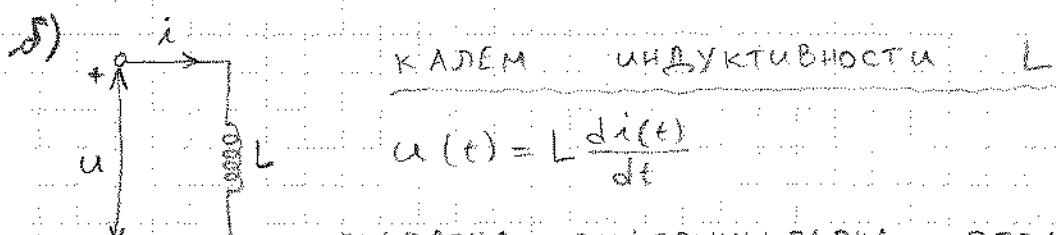
δ) ИНТЕНЗИТЕТ СТРУЈЕ ПРИ НАЈВСНОЈ БРЗИНИ ПРОМЕЊЕ

JE НУДА!

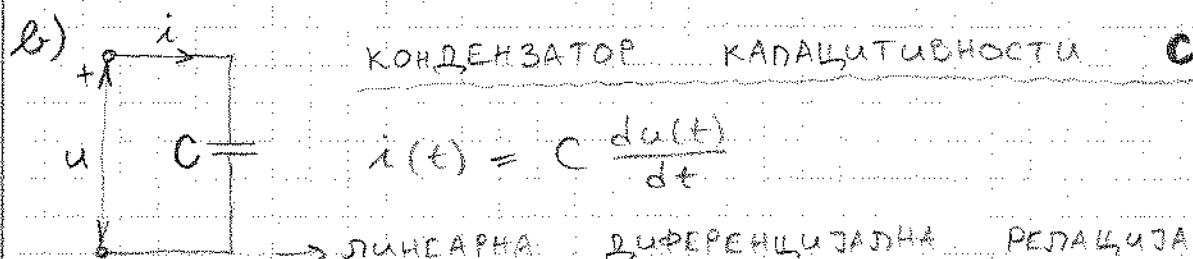
2. ЕЛЕМЕНТИ КОЈА ПРОМЕЊИВИХ СТРУЈА



→ ЛИНЕАРНА АДДЕВАРСКА РЕЛАЦИЈА (линеарна пропорционалност)



→ ЛИНЕАРНА ДИФЕРЕНЦИЈАЛНА РЕЛАЦИЈА



→ ЛИНЕАРНА ДИФЕРЕНЦИЈАЛНА РЕЛАЦИЈА

52. РЕФЕРЕНТНИ СМЕРОВИ НАПОНА И СТРУЈЕ СУ НЕУСКЛАДЕНИ:

1° отпорник отпорности R :

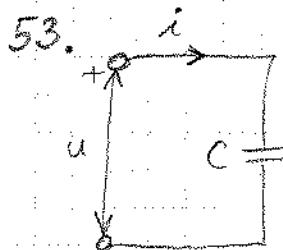
$$u(t) = -R \cdot i(t)$$

2° кајда индуктивности L :

$$u(t) = -L \frac{di(t)}{dt}$$

3° кондензатор капацитивности:

$$i(t) = -C \frac{du(t)}{dt}$$



$$u(t) = U\sqrt{2} \cos \left(\omega t - \frac{\pi}{6} \right)$$

a) ТРЕНУТНА СТРУЈА

$$i(t) = C \frac{du(t)}{dt} = C \frac{d}{dt} \left(U\sqrt{2} \cos \left(\omega t - \frac{\pi}{6} \right) \right)$$

$$i(t) = -\omega C U \sqrt{2} \sin \left(\omega t - \frac{\pi}{6} \right)$$

$$i(t) = \omega C U \sqrt{2} \cos \left(\omega t - \frac{\pi}{6} + \frac{\pi}{2} \right) = \boxed{\omega C U \sqrt{2} \cos \left(\omega t + \frac{\pi}{3} \right)}$$

ТРЕНУТНА СНАГА КОНДЕНЗАТОРА

$$p(t) = u(t) \cdot i(t)$$

$$p(t) = U\sqrt{2} \cos \left(\omega t - \frac{\pi}{6} \right) \cdot \left(-\omega C U \sqrt{2} \sin \left(\omega t - \frac{\pi}{6} \right) \right)$$

$$p(t) = -\omega C U^2 \sin \left(\omega t - \frac{\pi}{6} \right) \cdot \cos \left(\omega t - \frac{\pi}{6} \right)$$

$$p(t) = -\omega C U^2 \sin \left(2\omega t - \frac{\pi}{3} \right) = \omega C U^2 \cos \left(2\omega t - \frac{\pi}{3} + \frac{\pi}{2} \right)$$

$$\boxed{p(t) = \omega C U^2 \cos \left(2\omega t + \frac{\pi}{6} \right)}$$

СРЕДЊА СНАГА

$$P_{sr} = 0$$

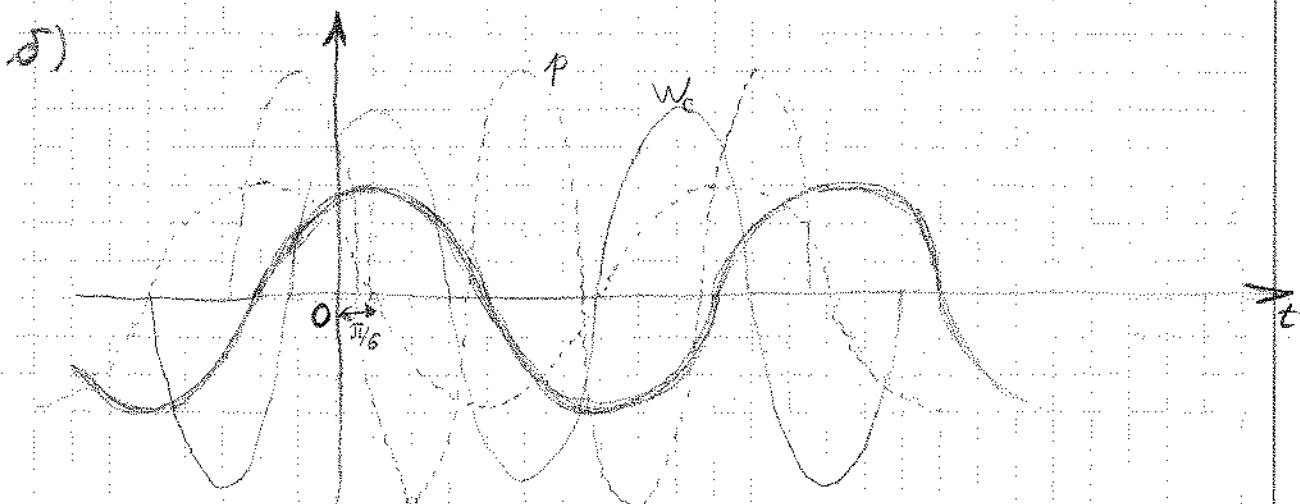
ТРЕНУТНА ЕЛЕКТРИЧНА ЕНЕРГИЈА

$$W_C(t) = \frac{1}{2} C u^2(t) = \frac{1}{2} C U^2 \cdot \frac{1}{2} \cos^2 \left(\omega t - \frac{\pi}{6} \right)$$

$$\boxed{W_C(t) = C U^2 \cos^2 \left(\omega t - \frac{\pi}{6} \right)}$$

$$\text{MAX: } W_{C\max} = C U^2 = \frac{1}{2} C U_m^2, \quad U = \frac{U_m}{\sqrt{2}} \Leftrightarrow U_m = U\sqrt{2}$$

δ)



$$54. u(t) = U_m(1 + \cos \omega t), U_m, \omega = \text{const}$$

РЕЗИСТАНТНЫЙ СМЕРЖ УКАЗАНИЕ СА НА ПОНОМ

$$Q(t) = CU(t)$$

$$Q(t) = CU_m(1 + \cos \omega t)$$

$$i(t) = C \frac{dU(t)}{dt} = C \frac{d}{dt}(U_m(1 + \cos \omega t))$$

$$i(t) = -CU_m \omega \sin \omega t = -\omega CU_m \sin \omega t$$

$$i(t) = -CU_m \omega \sin \omega t$$

$$p(t) = u(t)i(t) = U_m(1 + \cos \omega t) \cdot (-\omega CU_m \sin \omega t)$$

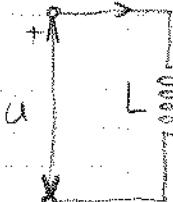
$$p(t) = -\omega CU_m^2 (1 + \cos \omega t) \cdot \sin \omega t$$

$$p(t) = -CU_m^2 \omega (\sin \omega t + \frac{1}{2} \sin 2\omega t)$$

$$P_{SR} = \frac{1}{T} \int_0^T p(t) dt = \frac{1}{T} \left(\int_0^T -CU_m^2 \omega \sin \omega t dt + \int_0^T \frac{1}{2} CU_m^2 \omega \sin 2\omega t dt \right)$$

$$P_{SR} = 0$$

$$55. i \quad \text{КАПЕМ ИНДУКТИВНОСТИ } L$$



$$i(t) = I_o + I_m \cos \omega t; I_o, I_m, \omega = \text{const}$$

$$u(t) = L \frac{di(t)}{dt}$$

$$u(t) = L \frac{d}{dt} (I_o + I_m \cos \omega t)$$

$$u(t) = L I_m (-\omega \sin \omega t) = -\omega L I_m \sin \omega t$$

$$u(t) = -L I_m \omega \sin \omega t$$

$$p(t) = u(t) \cdot i(t) = -L I_m \omega \sin \omega t \cdot (I_o + I_m \cos \omega t)$$

$$p(t) = -LI_o I_m \omega \sin \omega t - LI_m^2 \omega \frac{1}{2} \sin 2\omega t$$

$$p(t) = -L \omega (I_o I_m \sin \omega t + \frac{1}{2} I_m^2 \sin 2\omega t)$$

$$P_{SR} = \frac{1}{T} \int_0^T p(t) dt$$

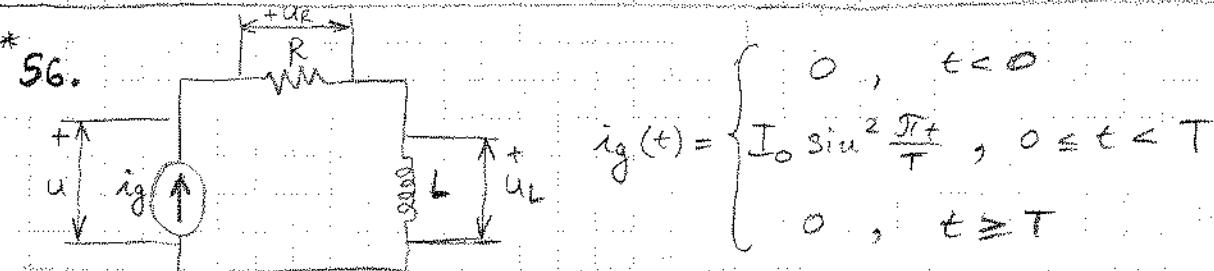
$$P_{SR} = 0$$

$$W_C(t) = \frac{1}{2} L i^2(t) = \frac{1}{2} L (I_0 + I_m \cos \omega t)^2$$

$$W_C(t) = \frac{1}{2} L (I_0 + I_m \cos \omega t)^2$$

УЗМЕДУ ЧАРГЕ И ЕНЕРГИЈЕ ПЕДАЛУЈА:

$$\left\{ \begin{array}{l} p(t) = \frac{d W_C(t)}{dt} \end{array} \right.$$



$$I_0 = 10 \text{ mA}$$

$$u_R = R \cdot i_g(t)$$

$$T = 2 \mu s$$

$$u_R = R I_0 \sin^2 \frac{\pi t}{T}$$

$$R = 100 \Omega$$

$$u_R(t) = R I_0 \sin^2 \frac{\pi t}{T}$$

$$L = 100 \mu H$$

$$u_L = L \frac{di_g(t)}{dt} = L \frac{d}{dt} (I_0 \sin^2 \frac{\pi t}{T})$$

$$u_L = L I_0 2 \left(\sin \frac{\pi t}{T} \right) \cdot \left(\cos \frac{\pi t}{T} \right) \frac{\pi}{T}$$

$$u_L(t) = L I_0 \frac{\pi}{T} \sin \frac{2\pi}{T} t$$

$$u_g(t) = u_R(t) + u_L(t) = R I_0 \sin^2 \frac{\pi t}{T} + L I_0 \frac{\pi}{T} \sin \frac{2\pi}{T} t$$

$$u_g(t) = R I_0 \sin^2 \frac{\pi t}{T} + L \frac{\pi I_0}{T} \sin \frac{2\pi}{T} t$$

$$p_R(t) = u_R(t) \cdot i_g(t) = R I_0 \sin^2 \frac{\pi t}{T} \cdot I_0 \sin^2 \frac{\pi t}{T}$$

$$p_R(t) = R I_0^2 \sin^4 \frac{\pi t}{T}$$

$$p_L(t) = u_L(t) \cdot i_g(t) = L I_0 \frac{\pi}{T} \sin \frac{2\pi}{T} t \cdot I_0 \sin^2 \frac{\pi t}{T}$$

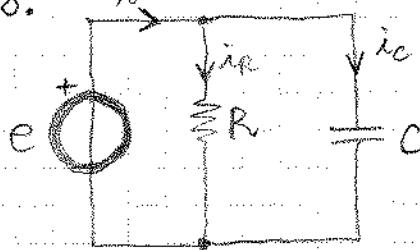
$$p_L(t) = L \frac{I_0^2 \pi}{T} \sin \frac{2\pi}{T} t \cdot \sin^2 \frac{\pi t}{T}$$

$$p_g(t) = u_g(t) \cdot i_g(t) = (R I_0 \sin^2 \frac{\pi t}{T} + L \frac{\pi I_0}{T} \sin \frac{2\pi}{T} t) (I_0 \sin^2 \frac{\pi t}{T})$$

$$p_g(t) = I_0^2 (R \sin^2 \frac{\pi t}{T} + L \frac{\pi}{T} \sin \frac{2\pi}{T} t) \sin^2 \frac{\pi t}{T} = [p_R(t) + p_L(t)]$$

** 57.

* 58. i



R, C

$i = ?$

$$e(t) = \begin{cases} 0, & t < 0 \\ E_0(1 - e^{-\frac{t}{\tau}}), & t \geq 0 \end{cases}, E_0, \tau = \text{const}$$

$$i_R(t) = \frac{e(t)}{R} = \begin{cases} 0, & t < 0 \\ \frac{E_0}{R}(1 - e^{-\frac{t}{\tau}}), & t \geq 0 \end{cases}$$

$$i_C(t) = C \frac{de(t)}{dt} = \begin{cases} 0, & t < 0 \\ \frac{E_0 C}{\tau} e^{-\frac{t}{\tau}}, & t \geq 0 \end{cases}$$

$$i(t) = i_R(t) + i_C(t) = \begin{cases} 0, & t < 0 \\ \frac{E_0}{R}(1 - e^{-\frac{t}{\tau}}) + \frac{E_0 C}{\tau} e^{-\frac{t}{\tau}}, & t \geq 0 \end{cases}$$

* СТРУЈА КОНДЕНЗАТОРА И СТРУЈА ГЕНЕРАТОРА УМАЈУ СКОК (ПРЕКИД) ЗА $t=0$!

** 59.

* 60. $R = 100 \Omega$

$$I_0 = 30A$$

$$\tau = 5 \mu s$$

I_{max}

16A

$$i(t) = \begin{cases} 0, & t < 0 \\ I_0(e^{\frac{t}{\tau}} - e^{-\frac{5t}{\tau}}), & t \geq 0 \end{cases}$$

$$A_J = ? \quad t \in (0, +\infty)$$

$$t_{max} = \frac{\tau}{4} \ln 5 \approx 2 \mu s$$

$$A_J = \int_0^{+\infty} R i^2(t) dt = \int_0^{+\infty} R \cdot I_0^2 (e^{\frac{t}{\tau}} - e^{-\frac{5t}{\tau}})^2 dt$$

$$A_J = RI_0^2 \left(\int_0^{+\infty} e^{-\frac{25t}{\tau}} dt - \int_0^{+\infty} 2e^{-\frac{6t}{\tau}} dt + \int_0^{+\infty} e^{-\frac{10t}{\tau}} dt \right)$$

$$A_J = RI_0^2 \left(-\frac{\tau}{2} (e^{-\frac{25t}{\tau}} + e^0) + 2 \cdot \left(-\frac{\tau}{3}\right) \cdot (e^{-\frac{6t}{\tau}} - e^0) + \left(\frac{\tau}{10}\right) \cdot (e^{-\frac{10t}{\tau}} + e^0) \right)$$

$$A_J = RI_0^2 \left(\frac{\tau}{2} - \frac{\tau}{3} + \frac{\tau}{10} \right) = RI_0 \frac{15 - 10 + 3}{30} \tau = \frac{4}{15} \tau RI_0^2 = \frac{4}{15} R I_0^2 \tau$$

3. ЈЕДНОСТАВНА КОЛА ПРОСТОПЕРИОДИЧНИХ СТРУЗА

3.1. ВРЕМЕНСКИ ДОМЕН

$$61. R = 100 \Omega$$

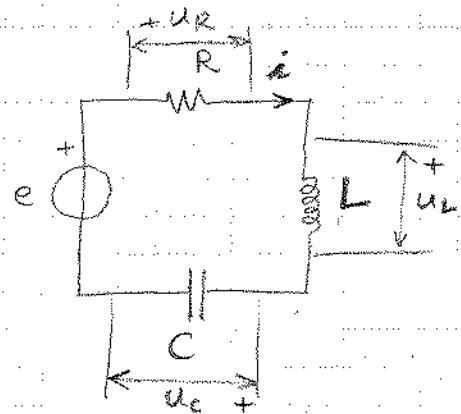
$$L = 10 \mu H$$

$$C = 500 \mu F$$

$$I_{\text{eff}} = I = 10 \text{ mA}$$

$$\omega = 10^7 \text{ s}^{-1}$$

$$\Psi = \frac{\pi}{3}$$



$$e(t) = ?$$

$$i(t) = I_m \cos(\omega t + \Psi) = I\sqrt{2} \cos(\omega t + \Psi)$$

$$u_R(t) = R \cdot i(t) = [RI\sqrt{2} \cos(\omega t + \Psi)]$$

$$u_L(t) = L \frac{di(t)}{dt} = -L\omega I\sqrt{2} \sin(\omega t + \Psi)$$

$$u_C(t) = \frac{1}{C} \int i(t) dt = \frac{1}{C} \int I\sqrt{2} \cos(\omega t + \Psi) dt$$

$$u_C(t) = \frac{1}{C} I\sqrt{2} \frac{1}{\omega} \sin(\omega t + \Psi) + U_0$$

$$u_C(t) = \frac{1}{\omega C} I\sqrt{2} \sin(\omega t + \Psi) + U_0$$

$$u_C(t) = \frac{1}{\omega C} I\sqrt{2} \sin(\omega t + \Psi - \frac{\pi}{2}) + U_0$$

II КИРХОФОВ ЗАКОН:

$$e(t) = u_R(t) + u_L(t) + u_C(t)$$

$$e(t) = RI\sqrt{2} \cos(\omega t + \Psi) + \omega L I\sqrt{2} \cos(\omega t + \Psi + \frac{\pi}{2})$$

$$+ \frac{1}{\omega C} I\sqrt{2} \sin(\omega t + \Psi - \frac{\pi}{2}) + U_0$$

$$e(t) = RI\sqrt{2} \cos(\omega t + \Psi) + I\sqrt{2} \left(\omega L - \frac{1}{\omega C} \right) \cos(\omega t + \Psi + \frac{\pi}{2})$$

$$e(t) = I\sqrt{2} (R \cos(\omega t + \Psi) - \frac{1}{\omega C} \sin(\omega t + \Psi))$$

$$e(t) = E\sqrt{2} \cos(\omega t + \theta)$$

$$\phi = \theta - \psi \rightarrow \text{ФАЗНА РАЗЛИКА}$$

$$\Rightarrow \theta = \phi + \psi$$

$$e(t) = E\sqrt{2} \cdot \cos(\omega t + \phi + \psi) = E\sqrt{2} (\cos(\omega t + \psi) \cdot \cos \phi - \sin(\omega t + \psi) \sin \phi)$$

$$e(t) = E\sqrt{2} (\cos(\omega t + \psi) \cos \phi - \sin(\omega t + \psi) \sin \phi)$$

$$\Rightarrow IR = E \cos \phi / \sqrt{2} \quad IX = E \sin \phi / \sqrt{2}$$

$$I^2 R^2 + I^2 X^2 = E^2 (\sin^2 \phi + \cos^2 \phi) = E^2$$

$$E^2 = I^2 (R^2 + X^2)$$

$$E = ZI \Rightarrow Z^2 I^2 = (R^2 + X^2) I^2$$

$$\Rightarrow Z = \sqrt{R^2 + X^2}$$

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$\phi = \arctg \frac{X}{R}, R > 0$$

$$\omega L = 10^7 \cdot 10^{-5} = 100 \Omega$$

$$\frac{1}{\omega C} = \frac{1}{5 \cdot 10^7 \cdot 10^{-10}} = 0.2 \cdot 10^3 \Omega = 200 \Omega$$

$$X = (100 - 200) \Omega = -100 \Omega \quad (\text{РЕДНА ВЕЗА С ПРЕТЕЖНО КАПАСИТИВНА})$$

$$Z = \sqrt{100^2 + 100^2} = \sqrt{100 \cdot 2} = 100\sqrt{2} \Omega$$

$$E = 100\sqrt{2} \cdot 10^{-2} V \Rightarrow E = \sqrt{2} V$$

$$\phi = \arctg \frac{-100}{100} = \arctg(-1) \Rightarrow \phi = -\frac{\pi}{4}$$

$$e(t) = \sqrt{2} \cdot \sqrt{2} \cos \left(\omega t + \frac{\pi}{3} - \frac{\pi}{4} \right) = 2 \cos \left(\omega t + \frac{4 - 3\pi}{12} \right)$$

$$e(t) = 2 \cos \left(\omega t + \frac{\pi}{12} \right) V$$

$$62. \quad G = 5 \text{ ms}$$

$$L = 10 \mu\text{H}$$

$$C = 500 \mu\text{F}$$

$$U_{12}(t) = U\sqrt{2} \cos(\omega t + \theta)$$

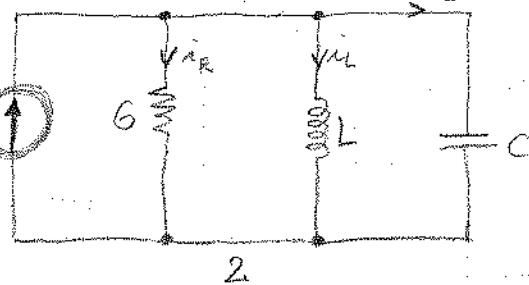
$$U = 1 \text{ V}$$

$$\omega = 10^7 \text{ s}^{-1}$$



i_g

ψ_R



ψ_L

$$\theta = \frac{\pi}{6}$$

$$i_g = ?$$

$$i_R(t) = \frac{U(t)}{R} = \frac{U\sqrt{2}}{R} \cos(\omega t + \frac{\pi}{6})$$

$$i_R(t) = GU\sqrt{2} \cos(\omega t + \theta)$$

$$i_L(t) = \frac{1}{L} \int u(t) dt = \frac{1}{L} \int GU\sqrt{2} \cos(\omega t + \theta) dt$$

$$i_L(t) = \frac{1}{L} GU\sqrt{2} \frac{1}{\omega} \sin(\omega t + \theta) + I_o$$

$$i_L(t) = \frac{1}{\omega L} GU\sqrt{2} \sin(\omega t + \theta) + I_o$$

$$i_L(t) = \frac{GU\sqrt{2}}{\omega L} \cos(\omega t + \theta - \frac{\pi}{2}) + I_o$$

$$i_C(t) = C \frac{dU(t)}{dt} = C \frac{d}{dt} (GU\sqrt{2} \cos(\omega t + \theta))$$

$$i_C(t) = GU\sqrt{2} (-\omega \sin(\omega t + \theta))$$

$$i_C(t) = -\omega C GU\sqrt{2} \sin(\omega t + \theta)$$

$$i_C(t) = \omega C GU\sqrt{2} \cos(\omega t + \theta + \frac{\pi}{2})$$

I KUPKOΦOB 3AKOTT:

$$i_g(t) = i_R(t) + i_L(t) + i_C(t)$$

$$i_g(t) = GU\sqrt{2} \cos(\omega t + \theta) + \frac{GU\sqrt{2}}{\omega L} \cos(\omega t + \theta - \frac{\pi}{2}) + I_o + \omega C GU\sqrt{2} \cos(\omega t + \theta + \frac{\pi}{2})$$

$$i_g(t) = GU\sqrt{2} \cos(\omega t + \theta) + GU\sqrt{2} \left(\omega C - \frac{1}{\omega L} \right) \cos(\omega t + \theta + \frac{\pi}{2})$$

$$B = \omega C - \frac{1}{\omega L} \Rightarrow \text{CYCLENTAHCA}$$

+ sin(\omega t + \theta)

$$i_g(t) = GU\sqrt{2} (G \cos(\omega t + \theta) - B \sin(\omega t + \theta))$$

$$i_g(t) = I\sqrt{2} \cos(\omega t + \varphi)$$

$$\gamma = \varphi - \theta = -\phi$$

$$\varphi = \gamma + \theta$$

$$i_g(t) = I\sqrt{2} \cos(\omega t + \gamma + \theta)$$

$$i_g(t) = I\sqrt{2} (\cos(\omega t + \theta) \cdot \cos \gamma - \sin(\omega t + \theta) \cdot \sin \gamma)$$

$$I \cdot \cos \gamma = U_G / 2 \quad I \cdot \sin \gamma = U_B / 2$$

$$\rightarrow I^2 (\sin^2 \gamma + \cos^2 \gamma) = U^2 (G^2 + B^2)$$

$$I^2 = U^2 (G^2 + B^2)$$

$$I = YU \Rightarrow Y^2 U^2 = U^2 (G^2 + B^2)$$

$$Y = \sqrt{G^2 + B^2}$$

$$\gamma = \arctan\left(\frac{B}{G}\right), G > 0$$

$$Y = \sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$\omega C = 10^7 \cdot 5 \cdot 10^{-10} = 5 \text{ mS}$$

$$\frac{1}{\omega L} = \frac{1}{10^2 \cdot 10^{-5}} = 10 \text{ mS}$$

$$B = 5 \text{ mS} - 10 \text{ mS} = -5 \text{ mS}$$

$$Y = \sqrt{25 + 25} = \sqrt{50} = \sqrt{25 \cdot 2}$$

$$Y = 5\sqrt{2} \text{ mS}$$

$$I = 5\sqrt{2} \text{ A}$$

$$\gamma = \arctan \frac{-5}{5} = \arctan(-1) = -\frac{\pi}{4} \Rightarrow \gamma = -\frac{\pi}{4}$$

$$i_g(t) = 5\sqrt{2} \cos\left(\omega t + \frac{\pi}{6} - \frac{\pi}{4}\right) = 5\sqrt{2} \cos\left(\omega t + \frac{2-\pi}{12}\right)$$

$$\{ i_g(t) = 5\sqrt{2} \cos\left(\omega t - \frac{\pi}{12}\right) \}$$

63. $R = 51 \Omega$

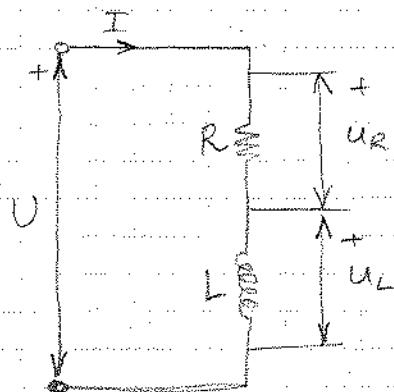
$L = 0.2 \text{ H}$

$u = U_m \cos(\omega t + \theta)$

$U_m = 155 \text{ V}$

$f = 50 \text{ Hz}$

$\theta = \pi$



a) $Z = ?$ PEREWE BEZEC

d) $I = ? \quad \psi = ?$

b) $U_R = ? \quad U_L = ?$

$U_R(t) = R \cdot i(t) = RI\sqrt{2} \cos(\omega t + \psi)$

$U_L(t) = L \frac{di(t)}{dt} = L \frac{d}{dt}(I\sqrt{2} \cos(\omega t + \psi))$

$U_L(t) = LI\sqrt{2}(-\omega \sin(\omega t + \psi))$

$U_L(t) = -\omega LI\sqrt{2} \sin(\omega t + \psi)$

$U_L(t) = \omega LI\sqrt{2} \cos(\omega t + \psi + \frac{\pi}{2})$

$u(t) = U_R(t) + U_L(t)$

$u(t) = RI\sqrt{2} \cos(\omega t + \psi) + \omega LI\sqrt{2} \cos(\omega t + \psi + \frac{\pi}{2})$

$u(t) = I\sqrt{2}(R \cos(\omega t + \psi) - \omega L \sin(\omega t + \psi))$

$u(t) = U\sqrt{2} \cos(\omega t + \theta)$

$\phi = \theta - \psi \Rightarrow \phi = \psi + \theta$

$u(t) = U\sqrt{2} \cos(\omega t + \phi + \psi)$

$u(t) = U\sqrt{2}(\cos(\omega t + \psi) \cdot \cos \phi - \sin(\omega t + \psi) \cdot \sin \phi)$

$IR = U \cos \phi / \sqrt{2} \quad I \omega L = U \sin \phi / \sqrt{2}$

$\rightarrow I^2(R^2 + (\omega L)^2) = U^2(\sin^2 \phi + \cos^2 \phi) = U^2$

$U = Z I \Rightarrow Z^2 I^2 = I^2(R^2 + (\omega L)^2)$

$Z = \sqrt{R^2 + (\omega L)^2}$

$$Z = \sqrt{R^2 + (2\pi f L)^2} \approx 81 \Omega$$

$$U = 2I \Rightarrow I = \frac{U}{Z} = \frac{110V}{81\Omega} = \frac{110V}{81\Omega} \Rightarrow I \approx 1.35A$$

$$\phi = \theta - \psi = 2\pi \operatorname{ctg} \frac{\omega L}{R} = 2\pi \operatorname{ctg} \frac{2\pi f L}{R}$$

$$\phi = 2\pi \operatorname{ctg} 1.23 \approx 51^\circ \approx 0.89 \text{ rad}$$

$$\Rightarrow \psi = \theta - \phi = \pi - 0.89 \approx 2.25 \text{ rad} \approx 123^\circ$$

$$i(t) = 1.35\sqrt{2} \cos(100\pi t + 2.25) A$$

b) $U_R = Z_R I = RI \approx 69.1 V$

$$U_L = Z_L I = \omega L I = 2\pi f L I \approx 85.1 V$$

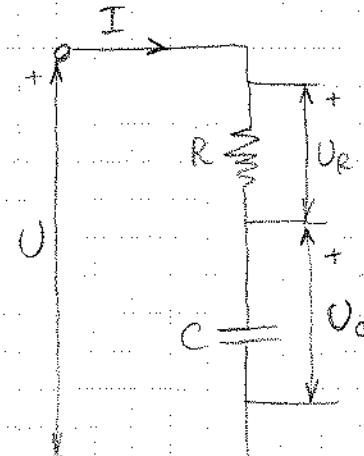
64. $R = 20 \Omega$

$$C = 15 \mu F$$

$$U = 110 V$$

$$f = 400 \text{ Hz}$$

$$\theta = 0$$



a) $i(t) = ?$

b) $U_R = ? \quad U_C = ?$

c) $\theta_C - \theta = ?$

d) $U_R(t) = R \cdot i(t) = R \sqrt{2} \cos(\omega t + \psi)$

$$U_C(t) = \frac{1}{C} \int i(t) dt = \frac{1}{C} \int \sqrt{2} \cos(\omega t + \psi) dt$$

$$U_C(t) = \frac{I\sqrt{2}}{C} \frac{1}{\omega} \sin(\omega t + \psi) = \frac{I\sqrt{2}}{\omega C} \sin(\omega t + \psi) + I_0$$

$$U(t) = U_R(t) + U_C(t)$$

$$U(t) = R \sqrt{2} \cos(\omega t + \psi) + \frac{I\sqrt{2}}{\omega C} \sin(\omega t + \psi) + I_0$$

$$U(t) = I\sqrt{2} \left(R \cos(\omega t + \psi) + \frac{1}{\omega C} \sin(\omega t + \psi) \right)$$

$$I_0 = 0$$

$$u(t) = UV_2 \cos(\omega t + \phi)$$

$$\phi = \theta - \Psi \Rightarrow \theta = \phi + \Psi$$

$$u(t) = UV_2 \cos(\omega t + \phi + \Psi)$$

$$u(t) = UV_2 (\cos(\omega t + \Psi) \cos \phi - \sin(\omega t + \Psi) \sin \phi)$$

$$IR = U \cos \phi / R \wedge I \frac{1}{\omega c} = U \sin \phi / R$$

$$\rightarrow I^2 (R^2 + \left(\frac{1}{\omega c}\right)^2) = U^2 (\sin^2 \phi + \cos^2 \phi) = U^2$$

$$U = ZI \Rightarrow Z^2 I^2 = I^2 (R^2 + \left(\frac{1}{\omega c}\right)^2)$$

$$Z^2 = R^2 + \left(\frac{1}{\omega c}\right)^2$$

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega c}\right)^2}$$

$$Z = \sqrt{R^2 + \left(\frac{1}{2\pi f c}\right)^2}$$

$$Z \approx 33.2 \Omega$$

$$I = \frac{U}{Z} = \frac{110V}{33.2\Omega} \Rightarrow I \approx 3.31A$$

$$\phi = \theta - \Psi$$

$$\Psi = \theta - \phi$$

$$i(t) = 3.31V_2 \cos ($$

$$\text{a)} U_R = Z_R \cdot I = RI \approx 66.2V$$

$$U_C = Z_C \cdot I = \frac{1}{\omega C} I = \frac{1}{2\pi f C} I \approx 87.8V$$

b)

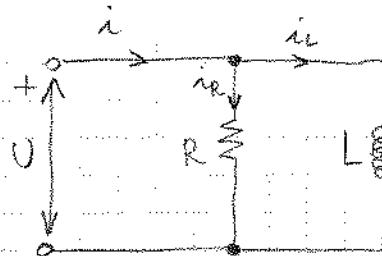
65. $R = 5\Omega$

$L = 20 \mu H$

$U = 220 V$

$f = 50 Hz$

$\omega = \frac{2\pi}{T}$



a) $I = ? \quad \varphi = ? \quad u(t) = U\sqrt{2} \cos(\omega t + \varphi)$

b) $I_R = ? \quad I_L = ?$

c) $\varphi_L - \varphi = ?$

a) $i_R(t) = \frac{u(t)}{R} = \frac{U\sqrt{2}}{R} \cos(\omega t + \varphi)$

$i_L(t) = \frac{1}{L} \int u(t) dt = \frac{1}{L} \int U\sqrt{2} \cos(\omega t + \varphi) dt$

$i_L(t) = \frac{U\sqrt{2}}{L} \frac{1}{\omega} \sin(\omega t + \varphi) = \frac{U\sqrt{2}}{\omega L} \sin(\omega t + \varphi)$

$i(t) = i_R(t) + i_L(t)$

$i(t) = \frac{U\sqrt{2}}{R} \cos(\omega t + \varphi) + \frac{U\sqrt{2}}{\omega L} \sin(\omega t + \varphi)$

$i(t) = I\sqrt{2} \cos(\omega t + \varphi)$

$\nu = \varphi - \theta \Rightarrow \varphi = \nu + \theta$

$i(t) = I\sqrt{2} \cos(\omega t + \nu + \theta)$

$i(t) = I\sqrt{2} (\cos(\omega t + \theta) \cdot \cos \nu - \sin(\omega t + \theta) \cdot \sin \nu)$

$\Rightarrow \frac{U}{R} = I \cos \nu / \sqrt{2} \quad \frac{U}{\omega L} = -I \sin \nu / \sqrt{2}$

$\rightarrow \frac{U^2}{R^2} + \frac{U^2}{(\omega L)^2} = I^2 (\sin^2 \nu + \cos^2 \nu) = I^2$

$I = \nu U \Rightarrow \nu^2 U^2 = U^2 \left(\left(\frac{1}{R} \right)^2 + \left(\frac{1}{\omega L} \right)^2 \right)$

$$\nu = \sqrt{\left(\frac{1}{R} \right)^2 + \left(\frac{1}{\omega L} \right)^2}$$

$\nu = \sqrt{\left(\frac{1}{R} \right)^2 + \left(\frac{1}{2\pi f L} \right)^2} \approx 256 \text{ mS}$

$$I = YU \approx 56,23 A$$

$$\vartheta = \arctan \frac{U}{I}$$

d) $I_R = Y_R U = \frac{U}{R} = \frac{220}{5} A \Rightarrow I_R = 44 A$

$$I_L = Y_L U = \frac{1}{2\pi f L} U \approx 35 A$$

$$\vartheta_L = \Psi_L - \vartheta = -\frac{\pi}{2} \quad (\text{предыдущий шаг } 3A - \frac{\pi}{2})$$

$$\vartheta = \Psi_L - \vartheta_L = \Psi - \vartheta$$

$$\vartheta = \Psi_L - \vartheta_L = \Psi - \vartheta$$

$$\Rightarrow \beta = \Psi_L - \Psi = \vartheta_L - \vartheta \Rightarrow \beta \approx -0.297 \text{ rad}$$

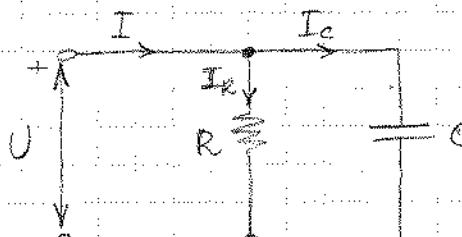
66. $R = 30 \Omega$

$$C = 5 \mu F$$

$$U = 110 V$$

$$f = 800 \text{ Hz}$$

$$\theta = \frac{\pi}{4}$$



a) $I_R = ? \quad I_C = ?$

a) $I_R = \frac{U}{R} = \frac{110 \text{ V}}{30 \Omega} \approx 3.67 \text{ A}$

d) $\vartheta - \vartheta_C = ?$

$$I_R = Y_R U$$

$$Y_C = i_C / U = 2\pi f C \approx 25.1 \text{ mS}$$

$$I_C = Y_C U \approx 2.76 \text{ A}$$

$$i_R(t) = \frac{U(t)}{R} = \frac{U\sqrt{2}}{R} \cos(\omega t + \theta)$$

$$i_C(t) = C \frac{dU(t)}{dt} = C \frac{1}{R} \left(U\sqrt{2} \cos(\omega t + \theta) \right)$$

$$i_C(t) = C U \sqrt{2} (-\omega \sin(\omega t + \theta))$$

$$i_C(t) = -\omega C U \sqrt{2} \sin(\omega t + \theta)$$

$$i(t) = i_C(t) + i_R(t)$$

$$i(t) = \frac{U\sqrt{2}}{R} \cos(\omega t + \varphi) - \omega C U \sqrt{2} \sin(\omega t + \theta)$$

$$i(t) = U \sqrt{2} \left(\frac{1}{R} \cos(\omega t + \varphi) - \omega C \sin(\omega t + \theta) \right)$$

$$i(t) = I \sqrt{2} \cos(\omega t + \psi)$$

$$\nu = \varphi - \theta \Rightarrow \psi = \nu + \theta$$

$$i(t) = I \sqrt{2} \cos(\omega t + \nu + \theta)$$

$$i(t) = I \sqrt{2} (\cos(\omega t + \theta) \cdot \cos \nu - \sin(\omega t + \theta) \cdot \sin \nu)$$

$$\Rightarrow \frac{U}{R} = I \cos \nu / \sqrt{2} \quad U \omega C = I \sin \nu / \sqrt{2}$$

$$\xrightarrow{+} \frac{U^2}{R^2} + U^2 (\omega C)^2 = I^2 (\sin^2 \nu + \cos^2 \nu) = I^2$$

$$I = YU \Rightarrow YU^2 = U^2 \left(\frac{1}{R^2} + (\omega C)^2 \right)$$

$$Y = \sqrt{\left(\frac{1}{R}\right)^2 + (\omega C)^2} \approx 41.7 \text{ mS}$$

$$(I = YU \approx 4.59 \text{ A})$$

$$\delta) \nu = \varphi - \theta = \arctan\left(\frac{\omega C}{\frac{1}{R}}\right) = \arctan(\omega R C) \approx 32^\circ$$

$$\nu \approx 0.21\pi \text{ rad} \quad (\nu > 0 \text{ ОРЕТЕЖНО КАНАЛИЧИВАА})$$

$$\nu_c = \varphi_c - \theta = \frac{\pi}{2}$$

$$\Rightarrow \theta = \varphi - \nu = \varphi_c - \nu_c$$

$$\Rightarrow \varphi - \varphi_c = \nu - \nu_c = \nu - \frac{\pi}{2}$$

$$\varphi \approx -0.29\pi \text{ rad}$$

СТРУЗА I ФАЗНО ЗАОСТАДЕ 34 СТРУЗОМ I_C 3A

$$0.29\pi \text{ rad}$$

$$67. R = 4 \Omega$$

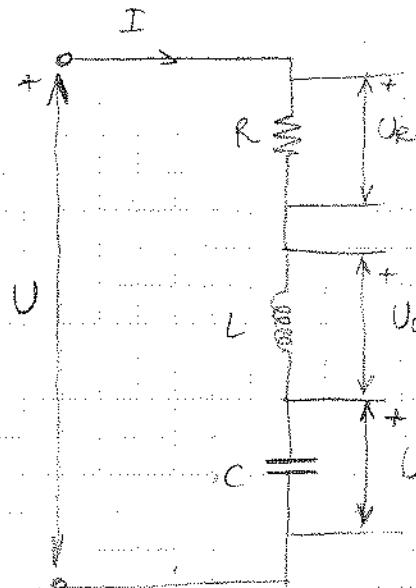
$$L = 10 \text{ mH}$$

$$C = 20 \mu\text{F}$$

$$U = 100 \text{ V}$$

$$\omega = 2512 \text{ s}^{-1}$$

$$\theta = -\frac{\pi}{2}$$



$$a) I = ? \quad \Phi = ?$$

$$b) U_R = ? \quad U_L = ? \quad U_C = ?$$

$$u_R(t) = R \cdot i(t) = R I \sqrt{2} \cos(\omega t + \Phi)$$

$$u_L(t) = L \frac{di(t)}{dt} = L \frac{d}{dt} (I \sqrt{2} \cos(\omega t + \Phi))$$

$$u_L(t) = L I \sqrt{2} (-\omega \sin(\omega t + \Phi))$$

$$u_C(t) = -\omega L I \sqrt{2} \sin(\omega t + \Phi)$$

$$u_C(t) = \frac{1}{C} \int i(t) dt = \frac{1}{C} \int I \sqrt{2} \cos(\omega t + \Phi) dt$$

$$u_C(t) = \frac{I \sqrt{2}}{C} \frac{1}{\omega} \sin(\omega t + \Phi) + U_0$$

$$u_C(t) = \frac{I \sqrt{2}}{\omega C} \sin(\omega t + \Phi) + U_0$$

$$u(t) = u_R(t) + u_L(t) + u_C(t)$$

$$u(t) = R I \sqrt{2} \cos(\omega t + \Phi) - \omega L I \sqrt{2} \sin(\omega t + \Phi) + \frac{I \sqrt{2}}{\omega C} \sin(\omega t + \Phi)$$

$$u(t) = I \sqrt{2} \left(R \cos(\omega t + \Phi) - \left(\omega L - \frac{1}{\omega C} \right) \sin(\omega t + \Phi) \right)$$

$$u(t) = U \sqrt{2} \cos(\omega t + \theta)$$

$$\theta = \Phi - \Psi \Rightarrow \theta = \Phi + \Psi$$

$$u(t) = U \sqrt{2} \cos(\omega t + \phi + \psi)$$

$$u(t) = U \sqrt{2} (\cos(\omega t + \Phi) \cdot \cos \phi - \sin(\omega t + \Phi) \cdot \sin \phi)$$

$$\Rightarrow IR = U \cos \phi / \sqrt{2} \quad I (\omega t + \frac{1}{\omega C}) = U \sin \phi / \sqrt{2}$$

$$I^2 R^2 + I^2 (\omega L - \frac{1}{\omega C})^2 = U^2 (\sin^2 \phi + \cos^2 \phi) = U^2$$

$$U = ZI \Rightarrow Z^2 I^2 = U^2 \left(R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right)$$

$$\boxed{Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}} \approx 6.57 \Omega$$

$$I = \frac{U}{Z} = \frac{100V}{6.57\Omega} \Rightarrow I \approx 15.2A$$

$$\omega L \approx 25.12 \Omega \quad \left\{ \begin{array}{l} (\omega L - \frac{1}{\omega C}) \approx 5.22 \Omega > 0 \\ \frac{1}{\omega C} \approx 19.9 \Omega \end{array} \right.$$

ВЕЗЯ ПРЕТЕЖНО ИНДУКТИВЧА!

$$\phi = \theta - \psi = \arctan \frac{\omega L - \frac{1}{\omega C}}{R} \approx 52.5^\circ \approx 0.23\pi \text{ rad} > 0$$

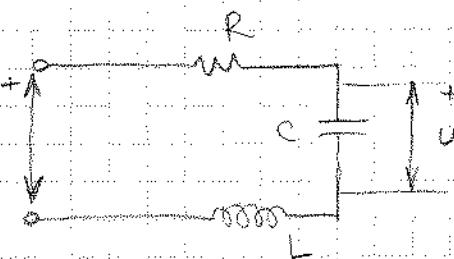
$$\Rightarrow \psi = \theta - \phi = -\frac{\pi}{2} - 0.23\pi = -0.73\pi \text{ rad} \approx -142.5^\circ$$

d) $U_R = 2_R I = RI \Rightarrow U_R \approx 60.3V$

$$U_L = 2_L I = \omega L I \Rightarrow U_L \approx 382V$$

$$U_C = 2_C I = \frac{I}{\omega C} \Rightarrow U_C \approx 303V$$

68.



R, L, C

$$U_C = U_C \sqrt{2} \cos(\omega t + \theta_C)$$

$$U(t) = U_R(t) + U_C(t) + U_L(t)$$

$$\phi_C = \theta_C - \psi = -\frac{\pi}{2} \Rightarrow \boxed{\psi = \theta_C + \frac{\pi}{2}}$$

$$I_C = \frac{U_C}{Z_C} = \frac{U_C}{\frac{1}{\omega C}} = \omega C U_C$$

$$i(t) = \omega C U_C \sqrt{2} \cos(\omega t + \theta_C + \frac{\pi}{2})$$

$$\phi = \theta - \psi = \arctan \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\Rightarrow \theta = \arctan \frac{\omega L - \frac{1}{\omega C}}{R} + \psi$$

$$\boxed{\theta = \theta_C + \frac{\pi}{2} + \arctan \frac{\omega L - \frac{1}{\omega C}}{R}}$$

$$u_R(t) = R i(t) = R I \sqrt{2} \cos(\omega t + \Psi)$$

$$u_L(t) = L \frac{di(t)}{dt} = L \frac{d}{dt} (I \sqrt{2} \cos(\omega t + \Psi))$$

$$u_L(t) = L I \sqrt{2} (-\omega \sin(\omega t + \Psi)) = -\omega L I \sqrt{2} \sin(\omega t + \Psi)$$

$$u_C(t) = \frac{1}{C} \int i(t) dt = \frac{1}{C} \int I \sqrt{2} \cos(\omega t + \Psi) dt$$

$$u_C(t) = \frac{1}{C} I \sqrt{2} \frac{1}{\omega} \sin(\omega t + \Psi) + I_0$$

$$u_C(t) = \frac{I \sqrt{2}}{\omega C} \sin(\omega t + \Psi)$$

$$u(t) = R I \sqrt{2} \cos(\omega t + \Psi) - \omega L I \sqrt{2} \sin(\omega t + \Psi) + \frac{I \sqrt{2}}{\omega C} \sin(\omega t + \Psi)$$

$$u(t) = I \sqrt{2} \left(R \cos(\omega t + \Psi) - \left(\omega L - \frac{1}{\omega C} \right) \sin(\omega t + \Psi) \right)$$

$$u(t) = U \sqrt{2} \cos(\omega t + \phi)$$

$$\phi = \theta - \Psi \Rightarrow \theta = \phi + \Psi$$

$$u(t) = U \sqrt{2} \cos(\omega t + \phi + \Psi)$$

$$u(t) = U \sqrt{2} (\cos(\omega t + \Psi) \cdot \cos \phi - \sin(\omega t + \Psi) \cdot \sin \phi)$$

$$\Rightarrow IR = U \cos \phi \quad \wedge \quad I \left(\omega L - \frac{1}{\omega C} \right) = U \sin \phi$$

$$I^2 R^2 + I^2 \left(\omega L - \frac{1}{\omega C} \right)^2 = U^2 (\sin^2 \phi + \cos^2 \phi) = U^2$$

$$U = Z I \Rightarrow Z^2 I^2 = I^2 (R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2)$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$$

$$U = \omega C U_c \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$$

$$u(t) = \omega C U_c \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2} \cos \left(\omega t + \theta_c + \frac{\pi}{2} + 2 \arctan \frac{\omega L - \frac{1}{\omega C}}{R} \right)$$

69. R, L, C

РЕДНА RLC ВЕЗА

a) $f = ?$ Z_{\min}

$Z \rightarrow$ индуканца

РЕДНА

ВЕЗА

d) $Z_{\min} = ?$

$$a) Z = \sqrt{R^2 + (wL - \frac{1}{wC})^2}$$

$$Z_{\min} \rightarrow wL - \frac{1}{wC} = 0 \Leftrightarrow wL = \frac{1}{wC} \Leftrightarrow w^2 LC = 1$$

$$d) \Rightarrow \boxed{Z_{\min} = R}$$

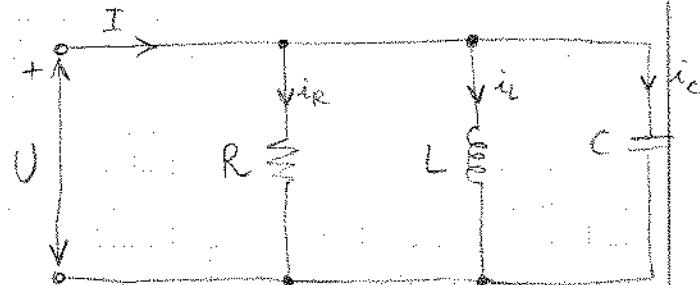
$$\boxed{f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}}$$

$$\Leftrightarrow \omega = \frac{1}{\sqrt{LC}}$$

70. R, L, C

$$u(t) = UV_2 \cos(\omega t + \delta)$$

$$i(t) = ?$$



$$i_R(t) = \frac{u(t)}{R} = \frac{UV_2}{R} \cos(\omega t + \delta)$$

$$i_L(t) = \frac{1}{L} \int u(t) dt = \frac{1}{L} \int UV_2 \cos(\omega t + \delta) dt$$

$$i_L(t) = \frac{1}{L} UV_2 \frac{1}{\omega} \sin(\omega t + \delta) = \frac{UV_2}{\omega L} \sin(\omega t + \delta)$$

$$i_C(t) = C \frac{du(t)}{dt} = C \frac{d}{dt} (UV_2 \cos(\omega t + \delta))$$

$$i_C(t) = CUV_2 (-\omega \sin(\omega t + \delta)) = -wCUV_2 \sin(\omega t + \delta)$$

$$i(t) = i_R(t) + i_L(t) + i_C(t)$$

$$i(t) = \frac{UV_2}{R} \cos(\omega t + \delta) + \frac{UV_2}{\omega L} \sin(\omega t + \delta) - wCUV_2 \sin(\omega t + \delta)$$

$$i(t) = UV_2 \left(\frac{1}{R} \cos(\omega t + \delta) + \left(wC - \frac{1}{\omega L} \right) \sin(\omega t + \delta) \right)$$

$$i(t) = IV_2 \cos(\omega t + \psi)$$

$$\psi = \psi - \delta \Rightarrow \boxed{\psi = \varphi + \delta}$$

$$i(t) = IV_2 \cos(\omega t + \gamma + \theta)$$

$$i(t) = IV_2 (\cos(\omega t + \theta) \cdot \cos \gamma - \sin(\omega t + \theta) \cdot \sin \gamma)$$

$$\Rightarrow \frac{U}{R} = I \cos \gamma \quad /^2 \quad U \left(\omega C - \frac{1}{\omega L} \right) = I \sin \gamma \quad /^2$$

$$\Rightarrow \frac{U^2}{R^2} + U^2 \left(\omega C - \frac{1}{\omega L} \right)^2 = I^2 (\sin^2 \gamma + \cos^2 \gamma) = I^2$$

$$I = \gamma U \Rightarrow \gamma^2 U^2 = U^2 \left(\left(\frac{1}{R} \right)^2 + \left(\omega C - \frac{1}{\omega L} \right)^2 \right)$$

$$\gamma = \sqrt{\left(\frac{1}{R} \right)^2 + \left(\omega C - \frac{1}{\omega L} \right)^2}$$

$$I = U \sqrt{\left(\frac{1}{R} \right)^2 + \left(\omega C - \frac{1}{\omega L} \right)^2}$$

$$\gamma = \Psi - \theta = \arctan \frac{\omega C - \frac{1}{\omega L}}{\frac{1}{R}} = \arctan (R \left(\omega C - \frac{1}{\omega L} \right))$$

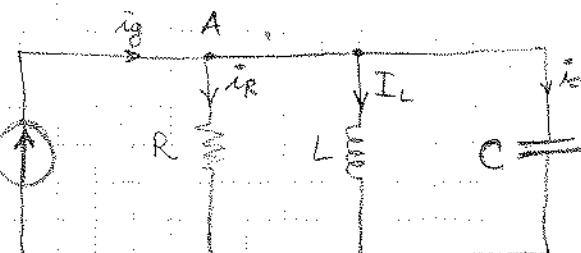
$$\Rightarrow \Psi = \theta + \arctan (R \left(\omega C - \frac{1}{\omega L} \right))$$

$$i(t) = UV_2 \sqrt{\left(\frac{1}{R} \right)^2 + \left(\omega C - \frac{1}{\omega L} \right)^2} \cos(\omega t + \theta + \arctan(R \left(\omega C - \frac{1}{\omega L} \right)))$$

71. R, L, C

$$i_L(t) = I_L V_2 \cos(\omega t + \Psi_L)$$

$$i_R(t) = ?$$



$$u_{AB}(t) = L \frac{di_L(t)}{dt} = L \frac{d}{dt} (I_L V_2 \cos(\omega t + \Psi_L))$$

$$u_{AB}(t) = L I_L V_2 (-\omega \sin(\omega t + \Psi_L))$$

$$u_R(t) = u_{AB}(t) = -\frac{\omega L I_L V_2}{R} \sin(\omega t + \Psi_L)$$

$$i_C(t) = C \frac{du_{AB}(t)}{dt} = C \frac{d}{dt} (-\omega L I_L V_2 \sin(\omega t + \Psi_L))$$

$$i_C(t) = -\omega C L I_L V_2 \cos(\omega t + \Psi_L)$$

$$U_{AB} = Z_L I_L = (\omega L I_L)$$

$$\begin{aligned} \theta_{AB} &= \theta_L \\ \phi_i &= \theta_L - \Psi_L \end{aligned} \quad \left. \begin{aligned} \theta_{AB} &= \theta_L + \Psi_L = \frac{\pi}{2} + \Psi_L \end{aligned} \right\}$$

$$Y = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$I_g = Y U_{AB} \Rightarrow I_g = \omega L I_L \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$\varphi_g = \Psi_g - \theta_{AB} \Rightarrow \varphi_g = \Psi_g + \theta_{AB}$$

$$\varphi_g = \arctan \frac{\omega C - \frac{1}{\omega L}}{\frac{1}{R}} = \arctan \left(R \left(\omega C - \frac{1}{\omega L} \right) \right)$$

$$\varphi_g = \frac{\pi}{2} + \Psi_L + \arctan \left(R \left(\omega C - \frac{1}{\omega L} \right) \right)$$

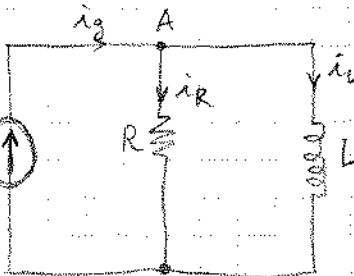
$$i_g(t) = I_g \sqrt{2} \cos(\omega t + \varphi_g)$$

$$\left. \begin{aligned} i_g(t) &= \omega L I_L \sqrt{2} \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2} \cos \left(\omega t + \Psi_L + \frac{\pi}{2} + \arctan \left(R \left(\omega C - \frac{1}{\omega L} \right) \right) \right) \end{aligned} \right\}$$

72. $R = 10 \Omega$

$$L = 6.4 \text{ mH}$$

$$i_g(t) = \frac{4.23}{I_{g_m}} \cos \left(\frac{3140}{\omega} t + \frac{\pi}{4} \right) A$$



$$U_{AB} = ?$$

$$I_{g_m} = 4.23 \Rightarrow I_g = \frac{I_{g_m} \sqrt{2}}{2}$$

$$\omega = 3140 \text{ s}^{-1}$$

$$\Psi_g = \frac{\pi}{4}$$

$$U_{AB} = U_{AB} \sqrt{2} \cos(\omega t + \theta_{AB})$$

$$i_R(t) = \frac{U_{AB}(t)}{R} = \frac{U_{AB} \sqrt{2}}{R} \cos(\omega t + \theta_{AB})$$

$$i_L(t) = \frac{1}{L} \int U_{AB}(t) dt = \frac{1}{L} \int U_{AB} \sqrt{2} \cos(\omega t + \theta_{AB}) dt$$

$$i_L(t) = \frac{U_{AB} \sqrt{2}}{\omega L} \sin(\omega t + \theta_{AB})$$

$$i_g(t) = i_R(t) + i_L(t)$$

$$i_g(t) = \frac{U_{AB}\sqrt{2}}{R} \cos(\omega t + \theta_{AB}) + \frac{U_{AB}\sqrt{2}}{\omega L} \sin(\omega t + \theta_{AB})$$

$$i_g(t) = U_{AB}\sqrt{2} \left(\frac{1}{R} \cos(\omega t + \theta_{AB}) + \frac{1}{\omega L} \sin(\omega t + \theta_{AB}) \right)$$

$$\gamma_g = \Psi_g - \theta_{AB} \Rightarrow \Psi_g = \gamma_g + \theta_{AB}$$

$$\bar{i}_g(t) = I_g\sqrt{2} (\cos(\omega t + \gamma_g + \theta_{AB}))$$

$$i_g(t) = I_g\sqrt{2} (\cos(\omega t + \theta_{AB}) \cdot \cos \gamma_g - \sin(\omega t + \theta_{AB}) \cdot \sin \gamma_g)$$

$$\Rightarrow \frac{U_{AB}}{R} = I_g \cos \gamma_g \quad \wedge \quad \frac{U_{AB}}{\omega L} = -I_g \sin \gamma_g$$

$$\rightarrow U_{AB}^2 \left(\left(\frac{1}{R} \right)^2 + \left(\frac{1}{\omega L} \right)^2 \right) = I_g^2 (\sin^2 \gamma_g + \cos^2 \gamma_g) = I_g^2$$

$$I_g = Y U_{AB} \Rightarrow Y^2 U_{AB}^2 = U_{AB}^2 \left(\left(\frac{1}{R} \right)^2 + \left(\frac{1}{\omega L} \right)^2 \right)$$

$$Y = \sqrt{\left(\frac{1}{R} \right)^2 + \left(\frac{1}{\omega L} \right)^2} = 0,412 \text{ S} = 112 \text{ mS}$$

$$U_{AB} = \frac{I_g}{Y} = \frac{\frac{I_g \omega L \sqrt{2}}{2}}{Y} = \frac{4,23}{0,412} \Rightarrow U_{AB} \approx 26,71 \text{ V}$$

$$\gamma_g = \arctan \frac{\frac{1}{\omega L}}{\frac{1}{R}} = \arctan \frac{R}{\omega L} \Rightarrow \gamma_g \approx 0,462 \text{ rad} \approx 0,15 \pi$$

$$\theta_{AB} = \Psi_g - \gamma_g = \frac{\pi}{4} - 0,462 \text{ rad} \Rightarrow \theta_{AB} \approx$$

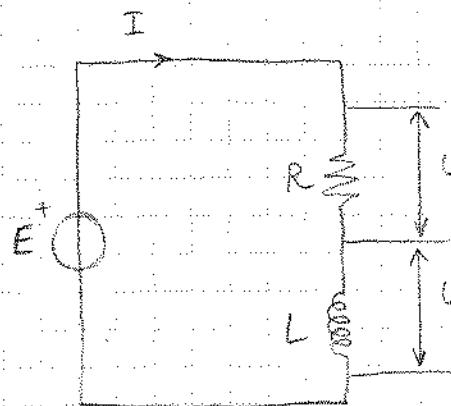
73. $R = 30 \Omega$

$$L = 50 \text{ mH}$$

$$e(t) = 191 \cos(800t[\text{s}] + \frac{\pi}{3}) \text{ V}$$

$$i(t) = ?$$

$$u_R(t), u_L(t); t_1 = \frac{\pi}{200} \text{ s}$$



$$e(t) = E\sqrt{2} \cos(800t[\text{s}] + \frac{\pi}{3})$$

$$E = 2I \Rightarrow (I = \frac{E}{2})$$

$$Z = \sqrt{R^2 + (\omega L)^2} = 50 \Omega \quad I = \frac{191 \text{ V}}{50 \Omega} \approx 2 \text{ A}$$

$$E = \frac{191}{\sqrt{2}} \approx 100 \text{ V}$$

$$\alpha = \frac{\pi}{3}$$

$$\phi = \theta - \psi = \arctan \frac{\omega L}{R} = \arctan \frac{\frac{1}{3}}{50} \approx 53^\circ \approx 0.295\pi \text{ rad} \quad (\text{известно})$$

$$\Rightarrow \psi = \alpha - \phi = \frac{\pi}{3} - 0.295\pi \Rightarrow \psi \approx 0.038\pi \text{ rad} \approx 6^\circ 52'$$

$$i(t) = 2\sqrt{2} \cos(800t[\text{s}] + 0.038\pi)$$

$$t_1 = \frac{\pi}{200} \text{ s}$$

$$i(t_1) \approx 2.8 \text{ A}$$

$$u_R(t_1) = R i(t_1) = 60\sqrt{2} \cos\left(\frac{\pi}{3} + 0.038\pi\right) \approx 84 \text{ V}$$

$$u_L(t_1) = L \frac{di(t_1)}{dt} = -\omega L I \sqrt{2} \sin(\omega t_1 + \psi)$$

$$u_L(t_1) = Z_L I \sqrt{2} \cos\left(\omega t_1 + \psi + \frac{\pi}{2}\right)$$

$$u_L(t_1) \approx -13.5 \text{ V}$$

74. $R = 5 \text{ k}\Omega$

$C = 1 \mu\text{F}$

$e(t) = 100 \sin(400t_{[s]}) \text{ V}$

$t_1 = \frac{\pi}{200} \text{ s} \quad 100 \cos(400t - \frac{\pi}{2})$

a) $i(t_1) = ?$

d) $u_R(t_1), u_C(t_1) = ?$

b) $p_e(t_1) = ?, p_R(t_1) = ?, p_C(t_1) = ?$

$$E\sqrt{2} = 100 \Rightarrow E = \frac{100}{\sqrt{2}} = \frac{100\sqrt{2}}{2} = 50\sqrt{2}$$

$$E = Z I \Rightarrow I = \frac{E}{Z} \quad I \approx 12.65 \text{ mA}$$

$$Z = \sqrt{R^2 + (\frac{1}{\omega C})^2} \approx 5.59 \text{ k}\Omega$$

$$\theta = -\frac{\pi}{2} \quad \psi = -\frac{\pi}{2}$$

$$\phi = \theta - \psi = \arctan \frac{1}{\omega C R} = \arctan \frac{1}{\omega CR} \quad \phi \approx$$

$$\phi \approx$$

$$i(t) = 12.65\sqrt{2} \cos(400t_{[s]})$$

d) $u_R(t_1) = R \cdot i(t_1)$

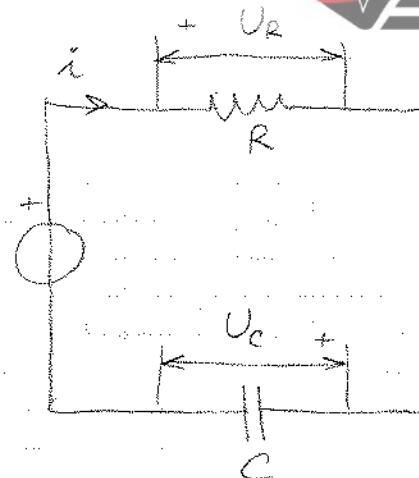
$$u_R(t_1) =$$

$$u_C(t_1) = Z_C i(t_1)$$

b) $p_R = u_R(t_1) \cdot i(t_1)$

$$p_e = u_C(t_1) \cdot i(t_1)$$

$$p_C = e(t_1) \cdot i(t_1)$$



$$75. u(t) = 50\sqrt{2} \cos(2000t[s] + \frac{\pi}{6}) V$$

$$i(t) = 5\sqrt{2} \cos(2000t[s] + \frac{\pi}{2}) A$$

$$R = ? \quad C = ?$$

$$U = 50 V$$

$$I = 5 A$$

$$\left. \begin{array}{l} U = 2I \\ I = 5 A \end{array} \right\} \Rightarrow$$

$$Z = \frac{U}{I} = \frac{50 V}{5 A} = 10 \Omega$$

$$Z = 10 \Omega$$

$$u(t)$$

$$i(t)$$

$$Z = \sqrt{R^2 + (\frac{1}{\omega C})^2}$$

$$\theta = \frac{\pi}{6}$$

$$\psi = \frac{\pi}{2}$$

$$\phi = \theta - \psi = \frac{\pi}{6} - \frac{\pi}{2} = -\frac{\pi}{3}$$

$$\phi = \arctan \frac{\frac{1}{\omega C}}{R} \Rightarrow \operatorname{tg} \phi = -\frac{1}{\omega R C}$$

$$WRC = \frac{1}{\omega C}$$

$$Z^2 = R^2 + \left(\frac{1}{\omega C}\right)^2 = 100$$

$$R^2 + 3R^2 = 100$$

$$R^2 = \frac{100}{4} = 25$$

$$R = 5 \Omega$$

$$\frac{1}{\omega C} = 5\sqrt{3} \Omega$$

$$\Rightarrow \frac{1}{C} = 5\sqrt{3} \cdot \omega = 5\sqrt{3} \cdot 2000$$

$$\frac{1}{C} = 10000\sqrt{3}$$

$$C \approx 57.7 \mu F$$

$$76. u(t) = 100 \cos(1000t[s]) V$$

$$i(t) = 10 \cos(1000t[s] + \frac{\pi}{3}) A$$

$$a) U_R = ? \quad U_C = ?$$

$$b) R = ? \quad C = ?$$

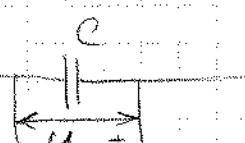
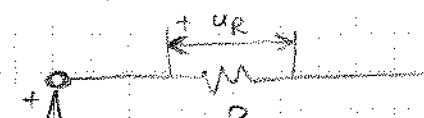
$$b) p_R^{(t)} = ? \quad p_C^{(t)} = ? \quad p(t_0) = ?$$

$$t_0 = \frac{\pi}{1000} s$$

$$\omega = 1000 \text{ rad/s}$$

$$U = 50\sqrt{2} V$$

$$I = 5\sqrt{2} A$$



a) $U_R = Z_R I = RI$

$$U_C = Z_C I = \frac{1}{\omega C} I$$

$$Z = \sqrt{R^2 + (\frac{1}{\omega C})^2} \Rightarrow z^2 = R^2 + \frac{1}{\omega^2 C^2}$$

$$z^2 = I^2 (R^2 + (\frac{1}{\omega C})^2)$$

$$U^2 = I^2 (R^2 + (\frac{1}{\omega C})^2)$$

$$U^2 = IR^2 + I^2 (\frac{1}{\omega C})^2 = U_R^2 + U_C^2 \Rightarrow U^2 = U_R^2 + U_C^2$$

$$\phi = \theta - \psi = \arctan \frac{-\frac{1}{\omega C}}{R} = -\frac{\pi}{3} \text{ rad} / \text{deg}$$

$$\tan(-\frac{\pi}{3}) = -\frac{1}{\omega CR} \Rightarrow \frac{1}{\omega CR} = \sqrt{3}$$

$$\phi = \arctan \left(\frac{-U_C}{U_R} \right) \Rightarrow \tan \theta = -\frac{U_C}{U_R} \Rightarrow \frac{U_C}{U_R} = \sqrt{3}$$

$$U^2 = U_R^2 (1 + 3) \Rightarrow U_R = \sqrt{\frac{(50\sqrt{2})^2}{4}} = \frac{50\sqrt{2}}{2}$$

$$U_R = 25\sqrt{2} \text{ V} \approx 35,4 \text{ V}$$

$$U_C = 25\sqrt{2} \cdot \sqrt{3} = 25\sqrt{6} \text{ V} \approx 61,2 \text{ V}$$

b) $U_R = Z_R I = RI \Rightarrow R = \frac{U_R}{I} = \frac{25\sqrt{2}}{25\sqrt{6}} = 5 \Omega$

$$U_C = Z_C I = \frac{1}{\omega C} I$$

$$R = 5 \Omega$$

$$\Rightarrow \omega C = \frac{I}{U_C} \Rightarrow C = \frac{I}{\omega U_C} = \frac{5\sqrt{2}}{1000 \cdot 25\sqrt{6}\sqrt{3}} = 0,115 \mu\text{F}$$

$$C = 115 \mu\text{F}$$

b) $p_R(t_1) = U_R(\epsilon_1) \cdot i(\epsilon_1)$

$$i(\epsilon_1) = 10 \cdot \cos \left(\pi + \frac{\pi}{3} \right) = 10 \cdot \cos \frac{4\pi}{3} = -5 \text{ A}$$

$$U_R(\epsilon_1) = 25\sqrt{2} \cdot \sqrt{2} \cos \left(\pi + \frac{\pi}{3} \right) = 50 \cdot \cos \frac{4\pi}{3} = -25 \text{ V}$$

$$p_R(t_1) = (-5) \cdot (-25) \text{ W} \Rightarrow \{ p_R(t_1) = 125 \text{ W} \}$$

$$b) p_c(t_1) = u_c(t_1) \cdot i(t_1)$$

$$u_c(t_1) = 25\sqrt{2} \cdot \sqrt{2} \cos(\pi + \theta_c)$$

$$\theta_c = \varphi - \frac{\pi}{2} = \frac{\pi}{3} - \frac{\pi}{2}$$

$$u_c(t_1) = 50\sqrt{3} \cos(\pi + \frac{\pi}{3} - \frac{\pi}{2}) = 50\sqrt{3} \cos(\frac{\pi}{2} + \frac{\pi}{3})$$

$$u_c(t_1) = 50\sqrt{3} \cos(\frac{5\pi}{6}) = 50\sqrt{3}(-\frac{\sqrt{3}}{2}) = -25\sqrt{3} V$$

$$\boxed{u_c(t_1) = -25\sqrt{3} V}$$

$$(p_c(t_1) = 725 \cdot (\sqrt{5}) W = 375 W)$$

$$p(t_1) = u(t_1) \cdot i(t_1) = p_R(t_1) + p_c(t_1)$$

$$p(t_1) = 125 W + 375 W \Rightarrow \boxed{p(t_1) = 500 W}$$

27. РЕДКА RC ОБИГА

$$E, \omega = \frac{1}{RC} \quad p_R(t) = u_R(t) \cdot i(t)$$

$$w_c = ? \quad p_{R \min} ! \quad p_R(t) = R \cdot i^2(t)$$

$$p_R(t) \text{ при } 3A \quad p_R(t) = 2Ri(t) = 0$$

$$w_c(t) = \frac{1}{2} C u_c(t) \quad (\Rightarrow i(t) = 0)$$

$$\Rightarrow \boxed{p_R(t) \text{ при } i=0}$$

$$\phi = \arctg \frac{w_c}{R} = -\arctg \frac{1}{w_c R} = -\arctg \frac{1}{\frac{1}{\omega} \cdot \omega} = -\arctg 1$$

$$\boxed{\phi = -\frac{\pi}{4}}$$

$$i(t) = 0 \rightarrow e(t) = u_c(t) = E\sqrt{2} \cos(\omega t + \phi)$$

$$\theta = \phi + \frac{\pi}{2} = \phi$$

$$\Rightarrow e(t) = E\sqrt{2} \cos \theta = E\sqrt{2} \cos(\frac{\pi}{4}) = E(-\frac{\sqrt{2}}{2})$$

$$\boxed{e(t) = u_c(t) = -E}$$

$$w_c(t) = \frac{1}{2} C (-E)^2 = \frac{1}{2} CE^2 \Rightarrow \boxed{w_c = \frac{1}{2} CE^2}$$

78. $e(t) = 100 \sin(400t_{[s]}) \text{ V}$

$$L = 25 \mu\text{H}$$

$$t_1 = \frac{\pi}{800} \text{ s}$$

$$U_L(t_1) = 40 \text{ V}$$

$$R = ?$$

$$E = 50\sqrt{2} \text{ V} \quad e(t) = (50\sqrt{2})\sqrt{2} \cos(400t_{[s]} - \frac{\pi}{2})$$

$$\theta = -\frac{\pi}{2}$$

$$i(t) = I\sqrt{2} \cos(\omega t + \varphi)$$

$$I = \frac{E}{Z}$$

$$i(t) = \frac{E}{Z}\sqrt{2} \cos(\omega t + \theta - \phi)$$

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$U_L(t_1) = L \frac{di(t)}{dt} = L \frac{d}{dt} \left(\frac{E}{Z} \sqrt{2} \cos(\omega t + \theta - \phi) \right)$$

$$U_L(t_1) = \frac{EL}{Z} \sqrt{2} (-\omega \sin(\omega t_1 + \theta - \phi))$$

$$U_L(t_1) = \omega \frac{EL}{Z} \sqrt{2} \cos(\omega t_1 + \theta - \phi + \frac{\pi}{2})$$

$$400 \cdot \frac{\pi}{800} = \frac{\pi}{2}$$

$$U_L(t_1) = \frac{\omega EL\sqrt{2}}{Z} \cos(\theta - \phi + \pi) = \frac{\omega EL\sqrt{2}}{Z} \cos(\frac{\pi}{2} - \phi)$$

$$U_L(t_1) = \frac{E\sqrt{2}}{Z} \omega L \sin(\phi) = \frac{E\sqrt{2}}{Z} \omega L \sin(\arctan \frac{\omega L}{R})$$

$$\sin^2 \alpha + \cos^2 \alpha = 1 \quad ; \quad \cos^2 \alpha = \frac{1}{1 + \tan^2 \alpha} \quad ; \quad \sin^2 \alpha = 1 - \frac{1}{1 + \tan^2 \alpha}$$

$$\tan^2 \alpha + 1 = \frac{1}{\cos^2 \alpha} \quad ; \quad 1 - \sin^2 \alpha = \frac{1}{1 + \tan^2 \alpha}$$

$$\sin^2 \alpha = \frac{\tan^2 \alpha}{1 + \tan^2 \alpha}$$

$$U_L(t_1) = \frac{E\sqrt{2}}{Z} \omega L \frac{\omega L}{\sqrt{1 + (\frac{\omega L}{R})^2}} = \frac{E\sqrt{2}}{Z} \omega L \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}$$

$$U_L(t_1) = \frac{E\sqrt{2}}{Z} \omega L \frac{\omega L}{2} = \frac{E\sqrt{2}}{Z} \left(\frac{\omega L}{2} \right)^2 = 40 \text{ V}$$

$$\left(\frac{\omega L}{2} \right)^2 = \frac{2 \cdot 40}{100} = \frac{2}{5} \Rightarrow \frac{(\omega L)^2}{4} = \frac{2}{5}$$

$$\omega L = 100 \cdot \frac{2}{5} = 40$$

$$\omega L = 10 \text{ rad/s}$$

$$\Rightarrow z^2 = \frac{100}{2} \Rightarrow z^2 = R^2 + 100 = 250 \Rightarrow R^2 = 150$$

$$R = \pm \sqrt{150} \Omega = \pm 12.25 \Omega$$

$\Rightarrow R = 12.25 \Omega$

79. $I = 0.2 \text{ A}$

$$\omega = 10^6 \text{ s}^{-1}$$

$$L = 10 \mu\text{H}$$

$$R = 20 \Omega$$

$$e(t) = ? \quad p(t) = ?$$

a) $i(t) = 0^+ /$

b) $i(t) = 0^- /$

c) $i(t)_{\max}$

d) $i(t)_{\min}$

a) $e(t) = u_L(t) + u_R(t)$

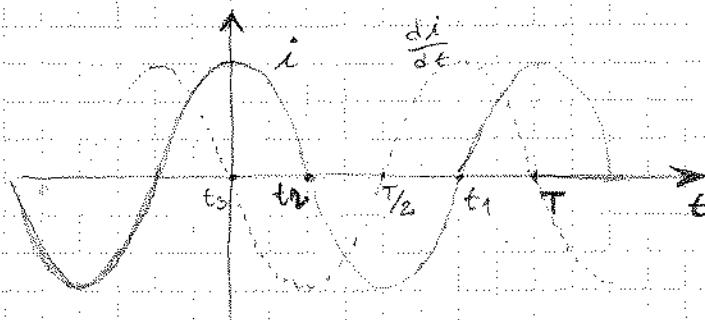
$$u_R(t) = R \cdot i(t)$$

$$u_L(t) = L \frac{di(t)}{dt}$$

$$i(t) = I\sqrt{2} \cos(\omega t + \phi) = I\sqrt{2} \cos \omega t$$

$$u_R(t) = R I \sqrt{2} \cos(\omega t)$$

$$u_L(t) = L \frac{d}{dt}(I\sqrt{2} \cos(\omega t)) = -w L I \sqrt{2} \sin(\omega t)$$



$$t = t_1 = \frac{3T}{4} \Rightarrow \omega t_1 = \frac{3\pi}{2}$$

$$e(t_1) = R I \sqrt{2} \cos \frac{3\pi}{2} + w L I \sqrt{2} \sin \frac{3\pi}{2} = w L I \sqrt{2}$$

$$e(t_1) = 10 \cdot 0.2 \sqrt{2} \text{ V} \Rightarrow e(t_1) = 2\sqrt{2} \text{ V}$$

$$p(t_1) = e(t_1) \cdot i(t_1) \Rightarrow p(t_1) = 0$$

d) $t_2 = \frac{T}{4} \Rightarrow \omega t_2 = \frac{\pi}{2}$

$$e(t_2) = RI\sqrt{2} \cos\left(\frac{\pi}{2}\right) - \omega L I\sqrt{2} \sin\left(\frac{\pi}{2}\right) = -\omega L I\sqrt{2}$$

$$e(t_2) = -10 \cdot 0.2\sqrt{2} V \Rightarrow e(t_2) = -2\sqrt{2} V \quad p(t_2) = e(t_2) \cdot i(t_2)$$

$$[p(t_2) = 0]$$

b) $i(t)_{\max} \rightarrow 3A \quad t_3 = 0$

$$e(t_3) = RI\sqrt{2} \cos(0) - \omega L I\sqrt{2} \sin(0) = RI\sqrt{2}$$

$$[e(t_3) = 4\sqrt{2} V] \quad p(t_3) = e(t_3) \cdot i(t_3) = 4\sqrt{2} \cdot 0.2\sqrt{2} \cos^2 0^\circ$$

$$p(t_3) = \frac{4}{5} \cdot 2 W = \frac{8}{5} W$$

$$(p(t_3) = 1.6 W)$$

c) $i(t)_{\min} \rightarrow 3A \quad t_4 = \frac{T}{2} \Rightarrow \omega t_4 = \pi$

$$e(t) = RI\sqrt{2} \cos(\omega t_4) - \omega L I\sqrt{2} \sin(\omega t_4)$$

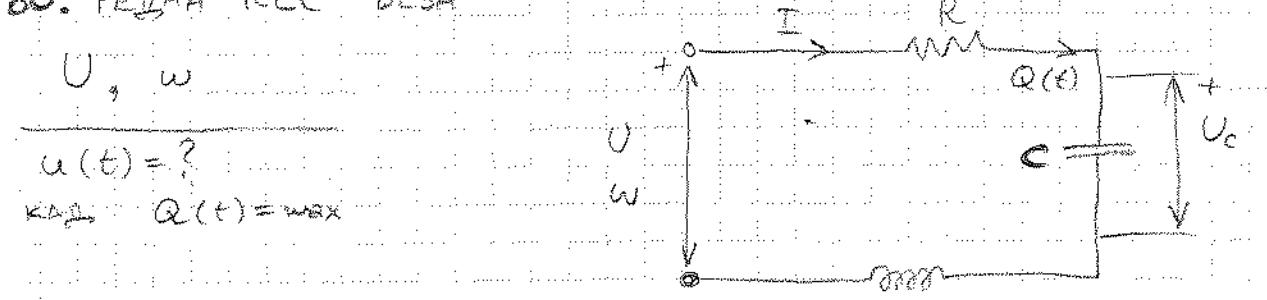
$$e(t_4) = RI\sqrt{2} \cos \pi - \omega L I\sqrt{2} \sin(\pi) = -RI\sqrt{2}$$

$$e(t_4) = -4\sqrt{2} V \quad p(t_4) = e(t_4) \cdot i(t_4)$$

$$p(t_4) = -4\sqrt{2} \cdot 0.2\sqrt{2} \cos \pi$$

$$p(t_4) = 0.8 \cdot 2 W \Rightarrow p(t_4) = 1.6 W$$

80. PERDIA RLC BEZA



$$Q(t) = C u_c(t) = C U_c \sqrt{2} \cos(\omega t + \theta_c)$$

$$Q(t) = Q_m \cos(\omega t + \theta_c) \quad \phi_c = \theta_c - \psi = -\frac{\pi}{2}$$

$$u(t) = U\sqrt{2} \cos(\omega t + \theta)$$

$$\theta = \theta_c - \psi$$

$$\psi = \theta_c + \frac{\pi}{2}$$

$$\Rightarrow \theta_c = \psi - \frac{\pi}{2}$$

$$Q(t) \text{ max} : 3A \quad \cos(\omega t_1 + \theta_c) = 0$$

$$\Leftrightarrow \omega t_1 + \theta_c = 2k\pi, \quad k=0, 1, 2, \dots$$

$$\omega t_1 + \Psi = \omega t_1 + \theta_c + \frac{\pi}{2} = \frac{\pi}{2}$$

$$\omega L = \frac{1}{\omega c}$$

$$\phi = \theta - \Psi = \arctan \frac{\omega L}{R}$$

$$\Rightarrow \Psi = \theta - \phi \quad \Leftrightarrow \theta = \phi + \Psi$$

$$u(t) = UV_2 \cos(\omega t_1 + \theta) = UV_2 \cos(\omega t_1 + \Psi + \phi)$$

$$u(t_1) = UV_2 \cos\left(\frac{\pi}{2} + \phi\right) = [-UV_2 \sin \phi]$$

$$u(t_1) = -UV_2 \sin\left(\arctan\left(\frac{\omega L}{R}\right)\right)$$

$$u(t_1) = -UV_2 \frac{\frac{\omega L}{R}}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}} = -UV_2 \frac{\frac{\omega L}{R}}{\sqrt{R^2 + (\omega L)^2}} = -UV_2 \frac{\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}}$$

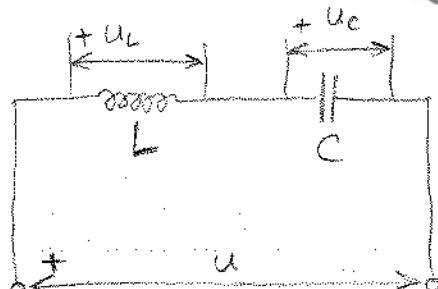
$$\left\{ u(t_1) = \frac{UV_2 \left(\frac{1}{\omega c} - \omega L\right)}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}}$$

$$84. u(t) = 20 \sin(\omega t) \text{ V}$$

$$\omega = 10^5 \text{ s}^{-1}$$

$$u_L(t) = 10 \cos\left(\omega t + \frac{\pi}{2}\right) \text{ V}$$

$$W_{C_{\max}} = 450 \mu\text{J}$$



$$L = ?$$

$$W_{C_{\max}} = \frac{1}{2} C U_{\max}^2$$

$$u(t) = 20 \cos\left(\omega t - \frac{\pi}{2}\right) \text{ V}$$

$$u(t) = u_L(t) + u_C(t)$$

$$\Rightarrow u_C(t) = u(t) - u_L(t)$$

$$u_C(t) = 20 \cos\left(\omega t - \frac{\pi}{2}\right) - 10 \cos\left(\omega t + \frac{\pi}{2}\right)$$

$$u_C(t) = 20 \cos\left(\omega t - \frac{\pi}{2}\right) + 10 \cos\left(\omega t - \frac{\pi}{2}\right)$$

$$u_C(t) = 30 \cos\left(\omega t - \frac{\pi}{2}\right)$$

$$\Rightarrow \boxed{U_{C_{\max}} = 30 \text{ V}}$$

$$\Rightarrow C = \frac{2 W_{C_{\max}}}{U_{C_{\max}}^2} = \frac{2 \cdot 450 \mu\text{J}}{300 \text{ V}^2} \text{ F} \Rightarrow \boxed{C = 1 \mu\text{F}}$$

$$\begin{aligned} U_{C_{\max}} &= Z_C I \\ U_{L_{\max}} &= Z_L I \end{aligned} \quad \left\{ \begin{aligned} U_{C_{\max}} &= \frac{Z_C I}{Z_L Z} = \frac{Z_C}{Z_L} = \frac{1}{\omega L} = \frac{30}{10} = \frac{3}{1} \\ U_{L_{\max}} &= \frac{Z_L I}{Z_L Z} = \frac{Z_L}{Z_L} = 1 \end{aligned} \right.$$

$$\Rightarrow \frac{1}{\omega L} = 3 \omega L \Rightarrow L = \frac{1}{3 \omega^2 C} = \frac{1}{3 \cdot 10^5 \cdot 10^{-9} \text{ s}^{-2}}$$

$$L = \frac{1}{30} \text{ mH} = \frac{100}{3000} \text{ mH} = \frac{100}{3} \mu\text{H}$$

$$\Rightarrow \boxed{L = \frac{100}{3} \mu\text{H}}$$

88. w

ПОЛУПРОВОДНИЧКА ДИОДА СЕ НАДАЗИ
У КОЈУ ПРОСТОПЕРИОДИЧНИХ ПОБУДА:

- ВЕЗА ИЗМЕЂУ НАПОНА И СТРУЈЕ ДИОДЕ ЈЕ НЕЛИНЕАРНА \rightarrow ПРОСТОПЕРИОДИЧАН РЕЖИМ НЕМОГУЋ
- АКО ДИОДУ ВЕЖЕМО НА ПРОСТОПЕРИОДИЧАН СТРУЈНИ ГЕНЕРАТОР, ВЕРНО НАПОН ЏЕ СЛОЖЕНОПЕРИОДИЧАН
- АКО ДИОДУ ВЕЖЕМО НА ПРОСТОПЕРИОДИЧАН НАПОЧСКИ ГЕНЕРАТОР, СТРУЈА ДИОДЕ ЏЕ СЛОЖЕНО ПЕРИОДИЧНА

89. КАДАМ СА ФЕРОМАГНЕТСКОМ ЈЕЗГРОМ -

ХИСТЕРЕЗИС ИЗРАНСК НАДАЗИ СЕ У КОЈУ ПРОСТОПЕРИОДИЧНЕ ПОБУДЕ;

- ФЕРОМАГНЕТСКО ЈЕЗГРО КАДЕМА је НЕЛИНЕАРНО $u = L \frac{di}{dt}$ ($L \neq \text{const}$)

ПРОСТОПЕРИОДИЧАН НАПОН \rightarrow СЛОЖЕНО ПЕРИОДИЧНА СТРУЈА

ПРОСТОПЕРИОДИЧНА СТРУЈА \rightarrow СЛОЖЕНОПЕРИОДИЧАН НАПОН

3.2. ФАЗОРСКИ ДИЈАГРАМИ

90. $E = 100V$

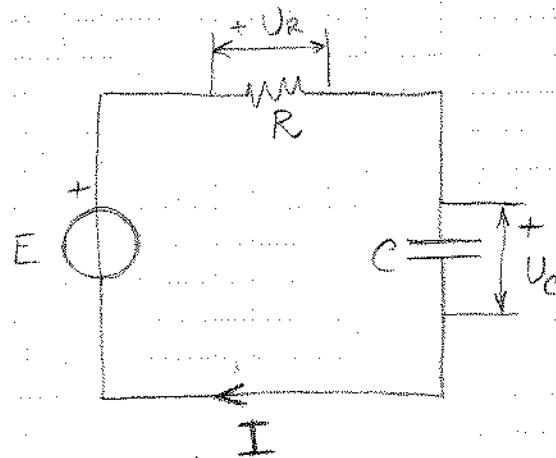
$$R = \frac{1}{\omega C} = 10\Omega$$

a) НАДРТАТИ ФАЗОРСКИ ДИЈАГРАМ

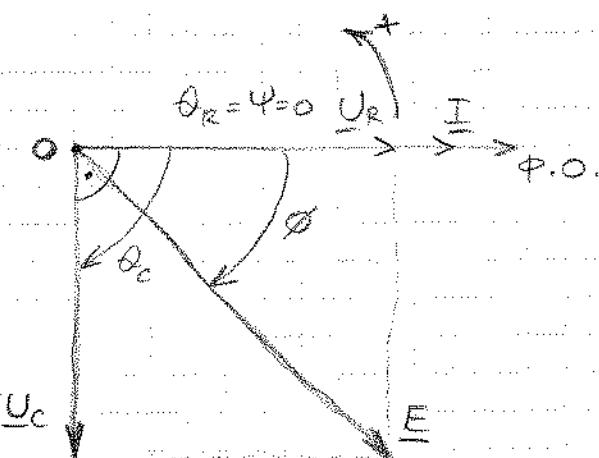
$$\delta) Z = ?$$

$$\phi = ?$$

$$U_R = ? \quad U_C = ?$$



a)



$$E = U_R + U_C$$

$$E^2 = U_R^2 + U_C^2$$

$$Z^2 I^2 = R^2 I^2 + \left(\frac{1}{\omega C}\right)^2 I^2$$

$$Z^2 I^2 = I^2 (R^2 + \left(\frac{1}{\omega C}\right)^2)$$

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\delta) \phi = \arctg \frac{-U_C}{U_R} = \arctg \frac{-\frac{1}{\omega C}}{R}$$

$$Z = \sqrt{10^2 + 10^2} = 10\sqrt{2}\Omega$$

$$\phi = \arctg \left(-\frac{1}{\omega CR} \right) = -\arctg \frac{1}{\omega C R} = -\arctg 1$$

$$\boxed{\phi = -\frac{\pi}{4}}$$

$$I = \frac{E}{Z} = \frac{100}{10\sqrt{2}} = \frac{10\sqrt{2}}{2} \Rightarrow \boxed{I = 5\sqrt{2} A}$$

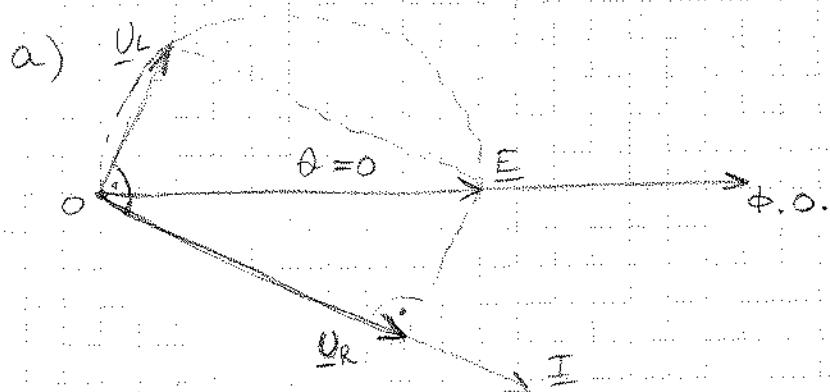
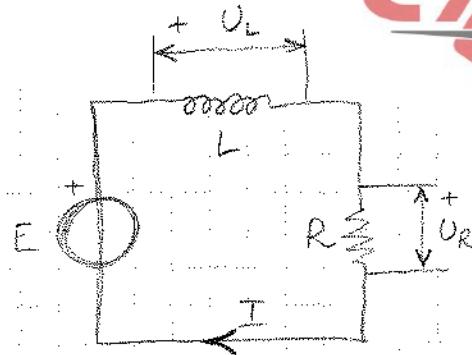
$$U_R = Z_R I = RI = 10 \cdot 5\sqrt{2} V \Rightarrow \boxed{U_R = 50\sqrt{2} V}$$

$$U_C = Z_C I = \frac{1}{\omega C} I = 10 \cdot 5\sqrt{2} V \Rightarrow \boxed{U_C = 50\sqrt{2} V}$$

$$\theta = 0$$

91. a) НАЈРТАТИ ФАЗОВСКИ
ДИЈАГРАМ

δ) $0 < L < +\infty$ (РМТ)

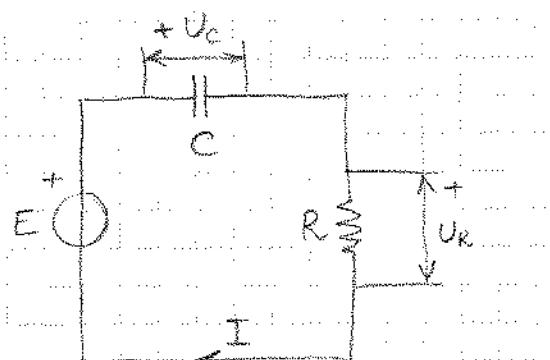


δ) ГЕОМЕТРИЈСКО МЕСТО (РМТ) ВРХОВА ФАЗОРА U_R

ЈЕ ПОЛУКРУГ КОНСТРУИСАН НАД ФАЗОРОМ E КАО ПРЕЧНИК.

92. a) НАЈРТАТИ ФАЗОВСКИ
ДИЈАГРАМ

δ) $0 < C < +\infty$ (РМТ)



δ) ГЕОМЕТРИЈСКО МЕСТО (РМТ) ВРХОВА ФАЗОРА U_C

ЈЕ ПОЛУКРУГ КОНСТРУИСАН НАД ФАЗОРОМ E КАО
ПРЕЧНИКОМ

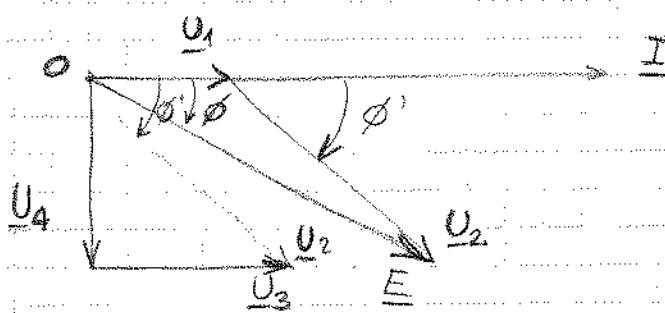
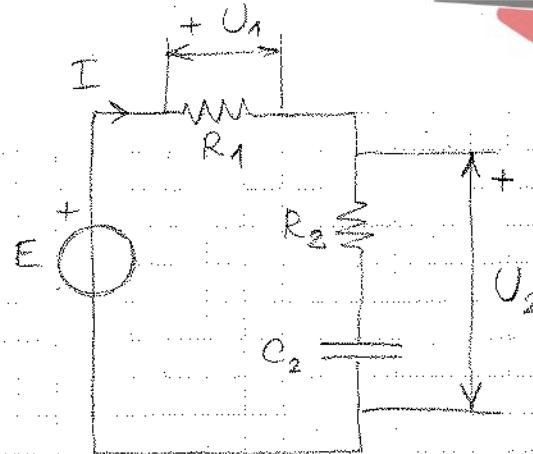
93. $E = 5V$

$$U_1 = 1V$$

$$U_2 = 2\sqrt{5} V$$

$$U_{R_2} = U_3 = ?$$

$$U_C = U_q = ?$$



$$\Delta U_1 U_2 E : E = U_1 + U_2$$

$$\text{КОСИНОУСКА ТЕОРЕМА : } E^2 = U_1^2 + U_2^2 - 2U_1 U_2 \cos(\pi - \phi')$$

$$E^2 = U_1^2 + U_2^2 + 2U_1 U_2 \cos \phi'$$

$$\Rightarrow \cos \phi' = \frac{E^2 - U_1^2 - U_2^2}{2U_1 U_2} = \frac{25 - 1 - 4 \cdot 5}{2 \cdot 1 \cdot 2\sqrt{5}} = \frac{25 - 21}{4\sqrt{5}} = \frac{4}{4\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\cos \phi' = \frac{\sqrt{5}}{5} \Rightarrow \sin^2 \phi' = 1 - \cos^2 \phi' = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\sin \phi' = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$\Delta U_2 U_3 U_q :$

$$\cos \phi' = \frac{U_3}{U_2} \Rightarrow U_3 = U_2 \cos \phi' = 2\sqrt{5} \cdot \frac{\sqrt{5}}{5} \Rightarrow U_3 = 2V$$

$$\sin \phi' = \frac{U_q}{U_2} \Rightarrow U_q = U_2 |\sin \phi'| = |2\sqrt{5} \cdot \frac{2}{\sqrt{5}}| \Rightarrow U_q = +4V$$

94. $U = 30\sqrt{5}$

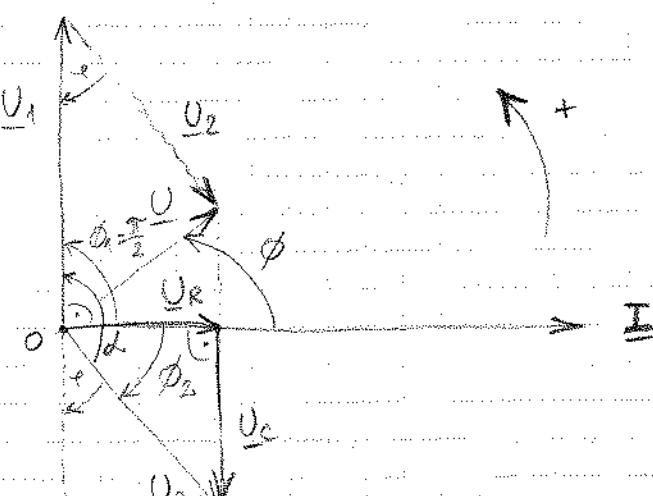
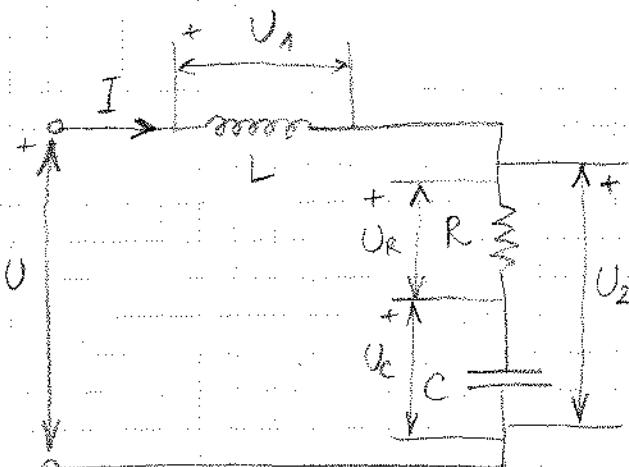
$$U_1 = 30V$$

$$U_2 = 30\sqrt{5}V$$

$$U_R = ?$$

$$U_C = ?$$

$$\phi = ?$$



$$\Delta U_1 U_2 V: \quad U_1 + U_2 = U$$

КОСИНУСНАЯ ТЕОРЕМА:

$$U^2 = U_1^2 + U_2^2 - 2U_1 U_2 \cos \phi \Rightarrow$$

$$U^2 = U_1^2 + U_2^2 - 2U_1 U_2 \cos(\pi - \alpha) = U_1^2 + U_2^2 + 2U_1 U_2 \cos \alpha$$

$$\Rightarrow 2U_1 U_2 \cos \alpha = \frac{U^2 - U_1^2 - U_2^2}{1} \Rightarrow \cos \alpha = \frac{U^2 - U_1^2 - U_2^2}{2U_1 U_2}$$

$$\cos \alpha = \frac{30^2 \cdot 2 - 3 \cdot 30^2 + 5 \cdot 30^2}{2 \cdot 3 \cdot 30 \cdot 30\sqrt{5}} = \frac{-2 \cdot 30^2}{16 \cdot 30^2 \sqrt{5}} \Rightarrow \cos \alpha = -\frac{2}{5\sqrt{5}}$$

$$\alpha = \frac{\pi}{2} - \phi_2 \quad (\phi_2 < 0)$$

$$\Rightarrow \cos \alpha = \cos\left(\frac{\pi}{2} - \phi_2\right) = \sin \phi_2 = -\frac{2}{5\sqrt{5}}$$

$$|\sin \phi_2| = \frac{U_C}{U_2} \Rightarrow U_C = U_2 |\sin \phi_2| = 30\sqrt{5} \cdot \frac{2}{5\sqrt{5}} \Rightarrow U_C = 60V$$

$$\sin \phi_2 = -\frac{2}{5\sqrt{5}} \Rightarrow \cos^2 \phi_2 = 1 - \sin^2 \phi_2 \Rightarrow \cos \phi_2 = \sqrt{1 - \frac{4}{25}} = \frac{1}{\sqrt{5}}$$

$$\cos \phi_2 = \frac{U_R}{U_2} \Rightarrow U_R = U_2 \cos \phi_2 = 30\sqrt{5} \cdot \frac{1}{\sqrt{5}} \Rightarrow \boxed{U_R = 30V}$$

ΔU_{UR} :

$$\cos \phi = \frac{U_R}{U} = \frac{30}{20\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \phi = \pm \arccos \frac{\sqrt{2}}{2} = \pm \frac{\pi}{4}$$

→ МРЕЖА СИНУСОДИЧЕСКАЯ:
 $(U_C < U_1)$

$$\phi = \frac{\pi}{4}$$

35. $R_1 = 5\Omega$

$R_2 = 20\Omega$

$U_C = 10\sqrt{3} V$

U_C ЗАОСТАЕТСЯ ЗА I_2

$$\alpha = \frac{\pi}{6}$$

a) $I = ?$

b) $\phi = \theta - \psi = ?$

$I = I_1 + I_2$

$U_{R2} = U_{R1} + U_C = U$

$U = U_{R2}$

$\phi_1 = \frac{\pi}{2} - \alpha = \frac{\pi}{2} - \frac{\pi}{6}$

$\Rightarrow \phi_1 = -\frac{\pi}{3}$

$\cos \alpha = \frac{U_C}{U} \Rightarrow U = \frac{U_C}{\cos \alpha}$

$U = \frac{10\sqrt{3}}{\frac{\sqrt{3}}{2}} \Rightarrow U = 20V$

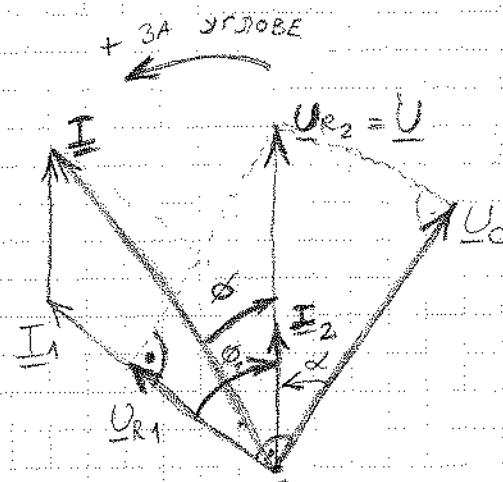
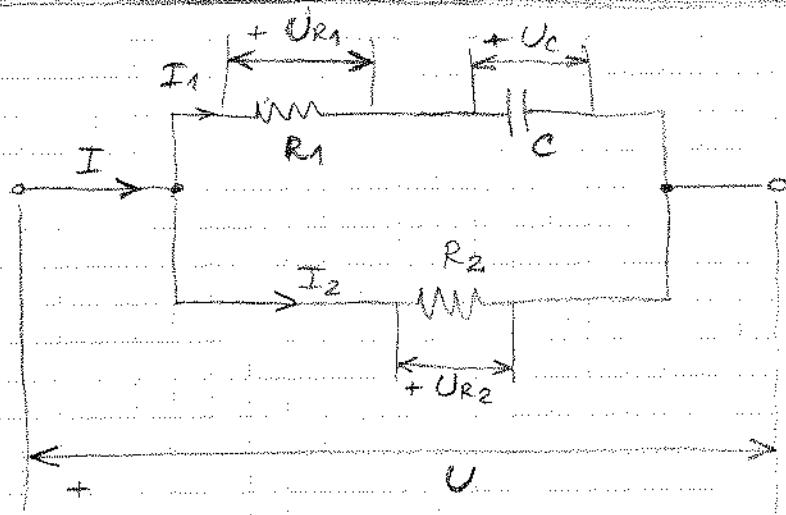
$\sin \alpha = \frac{U_{R1}}{U} \Rightarrow U_{R1} = U \sin \alpha = 20 \cdot \frac{1}{2} V \Rightarrow U_{R1} = 10V$

$U_{R2} = I_2 R_2 \Rightarrow I_2 = \frac{U_{R2}}{R_2} = \frac{U}{R_2} = \frac{20}{20} A \Rightarrow I_2 = 1A$

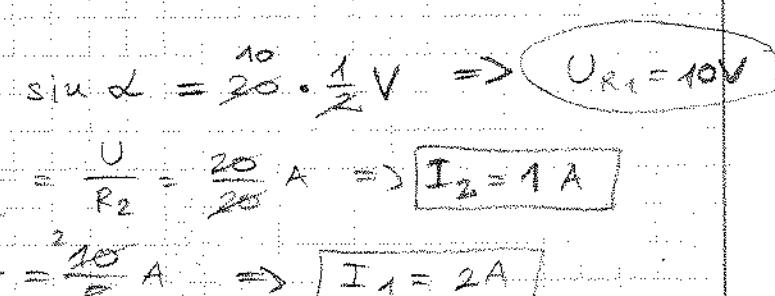
$U_{R1} = I_1 R_1 \Rightarrow I_1 = \frac{U_{R1}}{R_1} = \frac{10}{5} A \Rightarrow I_1 = 2A$

$\Delta I_1, I_2, I$:

$$I^2 = I_1^2 + I_2^2 - 2I_1 I_2 \cos(\pi - \phi_1) = I_1^2 + I_2^2 + 2I_1 I_2 \cos \phi_1$$



$$\alpha = \frac{\pi}{6}$$



$$I^2 = 4 + 1 + 2 \cdot 2 \cdot 1 \cdot |\cos \phi| = 5 + \frac{4}{2} = 7$$

$$I = \sqrt{7} \text{ A} \approx 2.65 \text{ A}$$

d) $\frac{I_1 \cdot I_2}{I^2} \cdot I :$

$$d) I_1^2 = I^2 + I_2^2 - 2 \cdot I \cdot I_2 \cos \phi$$

$$\Rightarrow \cos \phi = \frac{I^2 + I_2^2 - I_1^2}{2 \cdot I \cdot I_2} = \frac{7 + 1 - 4}{2 \cdot \sqrt{7} \cdot 1} = \frac{4}{2\sqrt{7}} = \frac{\sqrt{7}}{7}$$

$$\cos \phi = \frac{2}{\sqrt{7}}$$

$$\phi = \pm \arccos$$

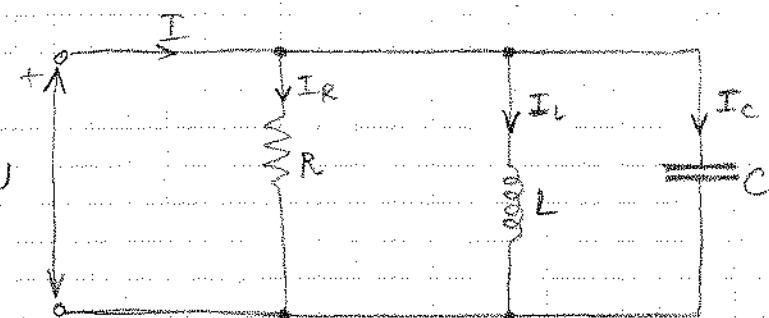
ОПРЕЖАЮЩАЯ КАПАСИТИВНА: $\phi \approx -$

96. $I = 5 \text{ A}$

$$I_L = 12.5 \text{ A}$$

$$I_C = 8.5 \text{ A}$$

$$I_R = ?$$



$$I_C$$

$$I_R + (I_L + I_C) = I$$

$$I_R = \frac{U}{R} \quad I^2 = I_R^2 + (I_L - I_C)^2$$

$$I_R^2 = I^2 - (I_L - I_C)^2$$

$$I_R = \sqrt{25 - (12.5 - 8.5)^2}$$

$$I_R = \sqrt{25 - 16} = \sqrt{9}$$

$$I_R = 3 \text{ A}$$

97. $U = \text{const}$

КРУЖНА УЧЕСТАНОСТ

ЗА ОД

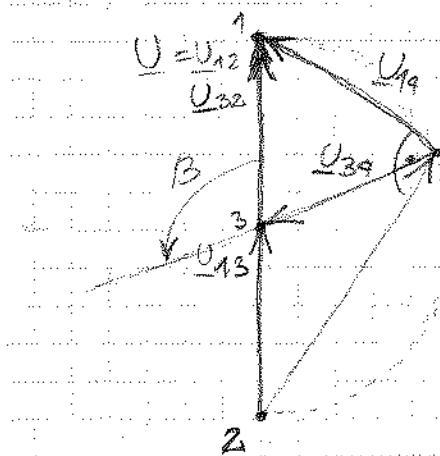
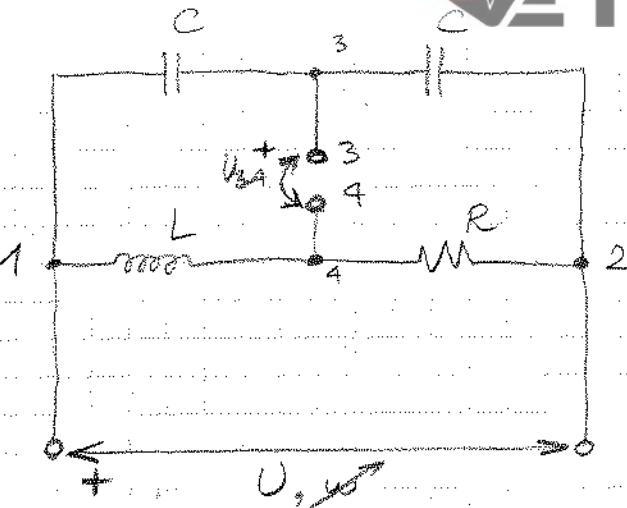
$$\omega_{\min} = \frac{R}{L\sqrt{3}}$$

ЗАТО

$$\omega_{\max} = \frac{RB}{L}$$

1) ПРИРАШТАЈ U_{34}

2) ПРИРАШТАЈ $\theta_{34} - \theta_{42}$



$$U_{13} = U_{32} = \frac{1}{2} U$$

$$\Rightarrow U_{13} = U_{32} = \frac{U}{2}$$

U_{14} ФАЗНО ПРЕДЊАЧИ U_{42} ЗА $\frac{\pi}{2}$

$$|U_{14} + U_{42}| = U$$

$$\Rightarrow U^2 = U_{14}^2 + U_{42}^2$$

- ПОЧЕТАК ФАЗОРА НАПОНА КАДЕМА U_{14} И ПОЧЕТАК

ФАЗОРА НАПОНА U_{34} СЕ ЗА СВЕ КРУЖНЕ УЧЕСТАНОСТИ

НАДАВЕ НА ПОЛУКРУГУ НАД ФАЗОРОМ НАПОНА U КООПРЕ.

$$\Rightarrow U_{34} = \frac{U}{2}$$

- ЕФЕКТИВНА ВРЕДНОСТ НАПОНА ЗАВИСИ ОД КРУЖНЕ УЧЕСТАНОСТИ ω , ЗАТО ЈЕ ПРИРАШТАЈ ЏЕЛНАК НУДИ.

$$\Delta U_{34} = U_{34}(\omega_{\max}) - U_{34}(\omega_{\min}) = 0$$

2) $\Delta U_{14} U_{34} U_{32}$: $U_{14} = U_{34} + U_{32}$

$$U_{14}^2 = U_{34}^2 + U_{32}^2 + 2 U_{34} U_{32} \cos(\beta - \alpha)$$

$$U_{14}^2 = U_{34}^2 + U_{32}^2 + 2 U_{34} U_{32} \cos \beta$$

$$\cos \beta = \frac{U_{14}^2 - U_{34}^2 - U_{32}^2}{2 U_{34} U_{32}}$$

$$\cos \beta = \frac{\frac{\omega L^2 U^2}{R^2 + (\omega L)^2} - \frac{U^2}{4} - \frac{U^2}{4}}{2 \cdot \frac{U}{2} \cdot \frac{U}{2}} = \frac{2}{(R^2 + \omega L)^2} - 1$$

$$U_{14} = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}} U$$

НАПОНСКИ ПРЕДЊАЧИК

* ПРИ ПРОМЕНИ КРУЖНЕ УЧЕСТАНОСТИ

$$B = \pi \text{ за } \omega = 0 \rightarrow B = 0 \text{ за } \omega = +\infty$$

$$3A : \omega \text{ от } \omega_{\min} = \frac{R}{L\sqrt{3}} \text{ до } \omega_{\max} = \frac{R\sqrt{3}}{L}$$

$$\cos B(\omega_{\min}) = \frac{2}{1 + \left(\frac{R}{L\sqrt{3}}\right)^2 - 1} = \frac{2}{4+3} - 1 = \frac{2}{9} - 1 = -\frac{1}{2}$$

$$B(\omega_{\min}) = \arccos\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$$

$$\cos B(\omega_{\max}) = \frac{2}{1 + \left(\frac{R\sqrt{3}}{L}\right)^2 - 1} = \frac{2}{1+\frac{1}{3}} - 1 = \frac{2}{\frac{4}{3}} - 1 = \frac{1}{2}$$

$$B(\omega_{\max}) = \arccos\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{3}}$$

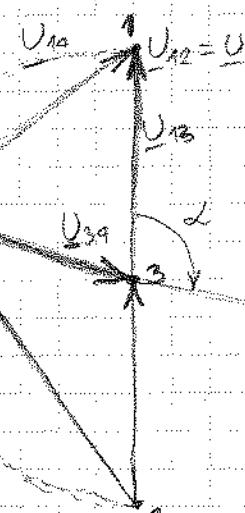
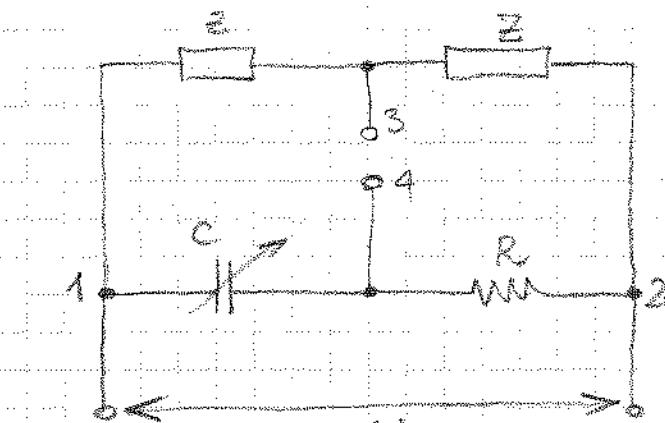
$$\Delta B = B(\omega_{\max}) - B(\omega_{\min}) = \frac{\pi}{3} - \frac{2\pi}{3} \Rightarrow \boxed{\Delta B = -\frac{\pi}{3}}$$

98. $C_{\min} = \frac{1}{wR\sqrt{3}}$

$$C_{\max} = \frac{\sqrt{3}}{wR}$$

$$\Delta d = d(C_{\max}) - d(C_{\min})$$

$$\phi(U_{34}) - \phi(U=U_{12})$$



$$U_{34} = \frac{U}{2}, U_{13} = \frac{U}{2}$$

$\Delta U_{12} U_{34} U_{13}$: косинусная троекома

$$U_{12}^2 = U_{34}^2 + U_{13}^2 - 2U_{34}U_{13} \cos(\beta - \alpha)$$

$$U_{12}^2 = U_{34}^2 + U_{13}^2 + 2U_{34}U_{13} \cos\alpha$$

$$\cos\alpha = \frac{U_{12}^2 - U_{34}^2 - U_{13}^2}{2U_{34}U_{13}}$$

$$\cos\alpha = \frac{\left(\frac{1}{wC}\right)^2 U^2 - \frac{U^2}{4} - \frac{U^2}{4}}{2 \cdot \frac{U}{2} \cdot \frac{U}{2}}$$

$$= \frac{2}{(wCR)^2 + 1} - 1$$

$$U_{12} = \frac{1}{wC} U$$

$$\sqrt{R^2 + \left(\frac{1}{wC}\right)^2}$$

$$\cos\alpha = \frac{2}{w^2(CR)^2 + 1} - 1$$

$$\cos \Delta (\text{Cmin}) = \frac{2}{\frac{\omega^2 R^2 + 1}{\omega^2 R^2} + 1} - 1 = \frac{2}{\frac{1}{3} + \frac{3}{3}} - 1 = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\Delta (\text{Cmin}) = 2\pi \cos \left(\frac{\pi}{2} \right) = \boxed{\frac{\pi}{3}}$$

$$\cos \Delta (\text{Cmax}) = \frac{2}{\frac{\omega^2 R^2 + 3}{\omega^2 R^2} + 1} - 1 = \frac{2}{\frac{2}{3} + 1} - 1 = -\frac{1}{2}$$

$$\Delta (\text{Cmax}) = 2\pi \cos \left(-\frac{\pi}{2} \right) = \boxed{\frac{2\pi}{3}}$$

$$\Delta \Delta = \Delta (\text{Cmax}) - \Delta (\text{Cmin}) = \frac{2\pi}{3} - \frac{\pi}{3} \Rightarrow \Delta \Delta = \frac{\pi}{3}$$

99. $R_1, Z_L, \frac{I_1}{I_2} = k, k > 0$

a) НАЦРТАТИ ФАЗОВСКИ Д.

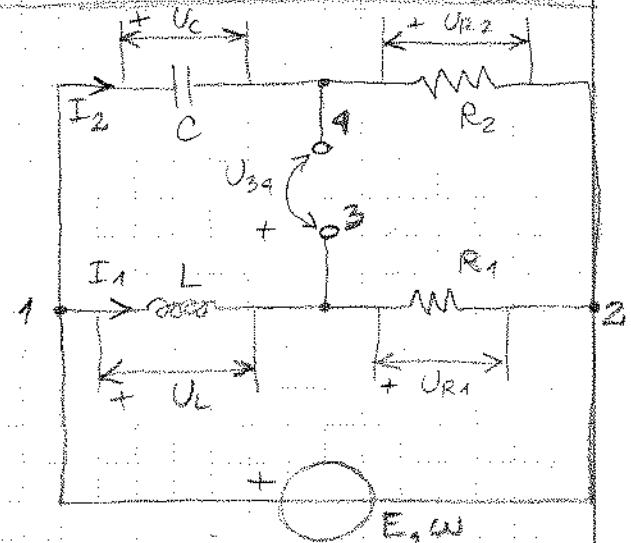
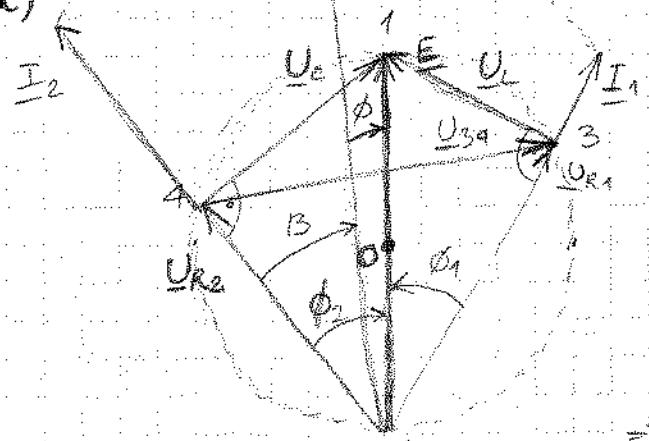
б) $R_2 = ?$; $Z_L = ?$

$U_{34} \rightarrow \text{макс}$

в) $U_{34\text{ макс}} = ?$

г) $\phi = \phi_1 - \phi_2 = ?$

д)



д) U_{34} је највећа кад

представља пречник круга који је конструисаног над E као пречником!

$$\Rightarrow U_{34} = E$$

$$U_{34} = U_{R_1} - U_{R_2}$$

$$\Rightarrow \phi_1 - \phi_2 = \frac{\pi}{2} \quad (U_{34} = E)$$

$$\Rightarrow \phi_1 = \frac{\pi}{2} + \phi_2 \quad (\phi_2 < 0)$$

$$\sin \phi_1 = \sin \left(\frac{\pi}{2} + \phi_2 \right) = \cos \phi_2$$

$$\cos \phi_1 = \cos \left(\frac{\pi}{2} + \phi_2 \right) = -\sin \phi_2$$

$$\frac{I_1}{I_2} = k \Rightarrow \frac{\frac{I_1}{Z_1}}{\frac{I_2}{Z_2}} = \frac{I_1}{I_2} \cdot \frac{Z_1}{Z_2} = \frac{1}{k} \Rightarrow \boxed{\frac{Z_2}{Z_1} = k}$$

$$\cos \phi_1 = \frac{U_{R1}}{E} = \frac{R_1 I_1}{E} = \frac{R_1 Z_1}{Z_1 Z_2} = \frac{R_1}{Z_2} \Rightarrow \boxed{\cos \phi_1 = \frac{R_1}{Z_2}}$$

$$|\sin \phi_2| = \frac{U_C}{E} = \frac{Z_2 Z_1}{Z_2 Z_1 + Z_1} = \frac{Z_1}{Z_2} \Rightarrow \boxed{\sin \phi_2 = -\frac{Z_1}{Z_2}}$$

$$\Rightarrow \cos \phi_1 = -\sin \phi_2 \Leftrightarrow \frac{R_1}{Z_2} = +\frac{Z_1}{Z_2} \Rightarrow \boxed{Z_C = \frac{Z_2}{Z_1} R_1}$$

$$\sin \phi_1 = \frac{U_L}{E} = \frac{Z_1 I_1}{Z_1 Z_2} + \frac{Z_L}{Z_1} \Rightarrow \boxed{\sin \phi_1 = \frac{Z_L}{Z_1}}$$

$$\cos \phi_2 = \frac{U_{R2}}{E} = \frac{R_2 I_2}{E} = \frac{R_2}{Z_2} \Rightarrow \boxed{\cos \phi_2 = \frac{R_2}{Z_2}}$$

$$\Rightarrow \sin \phi_1 = \cos \phi_2 \Leftrightarrow \frac{Z_L}{Z_1} = \frac{R_2}{Z_2} \Rightarrow R_2 = \frac{Z_2}{Z_1} Z_L \quad \boxed{R_2 = k Z_2}$$

b) ЕФЕКТИВНА ВРЕДНОСТ НАПОНА U_{34} : $\boxed{U_{34} = E}$

$$\begin{aligned} i) \quad \phi &= \theta - \psi \quad \left\{ \begin{array}{l} \phi = \theta - \psi = \phi_2 - \beta \\ \phi = \phi_2 - \beta \end{array} \right. \end{aligned}$$

$$\phi_2 = -\arccos \frac{R_2}{Z_2}$$

$$I_1 = k I_2, \quad I = \sqrt{I_1^2 + I_2^2} = \sqrt{k^2 I_2^2 + I_2^2} = \sqrt{I_2^2 (1+k^2)}$$

$$I = I_2 \sqrt{1+k^2}$$

$$\cos \beta = \frac{I_2}{I} = \frac{I_2}{I_2 \sqrt{1+k^2}} = \frac{1}{\sqrt{1+k^2}}$$

$$\beta = -\arccos \frac{1}{\sqrt{1+k^2}}$$

$$\phi = -\arccos \frac{R_2}{Z_2} + \arccos \frac{1}{\sqrt{1+k^2}}$$

$$100. \quad R_1 = 3\Omega$$

$$Z_L = 4\Omega$$

$$k = 2$$

$$E = 100V$$

$$R_2 = kZ_L = 2 \cdot 4\Omega \Rightarrow R_2 = 8\Omega$$

$$Z_C = kR_1 = 2 \cdot 3\Omega \Rightarrow Z_C = 6\Omega$$

$$R_2 = ?$$

$$Z_1 = \sqrt{Z_{R1}^2 + Z_L^2} = \sqrt{R_1^2 + Z_L^2}$$

$$Z_C = ?$$

$$Z_1 = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} \Omega$$

$$Z_1 = ?$$

$$Z_1 = 5\Omega$$

$$Z_2 = ?$$

$$Z_2 = \sqrt{Z_{R2}^2 + Z_C^2} = \sqrt{R_2^2 + Z_C^2}$$

$$Z = ?$$

$$Z_2 = \sqrt{8^2 + 6^2} = \sqrt{64+36} = \sqrt{100} \Omega$$

$$I_1 = ? \quad I_2 = ? \quad I = ?$$

$$Z_2 = 10\Omega$$

$$\phi = ?$$

$$\angle = \theta - \theta_{34} = ?$$

$$Z_{UL} = \frac{E}{I} = \frac{100}{10\sqrt{5}} = \frac{10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} \Omega$$

$$\Rightarrow Z_{UL} (= 2\sqrt{5}\Omega)$$

$$I_1 = \frac{E}{Z_1} = \frac{100}{5} A \Rightarrow I_1 = 20A$$

$$I_2 = \frac{E}{Z_2} = \frac{100}{10} A \Rightarrow I_2 = 10A$$

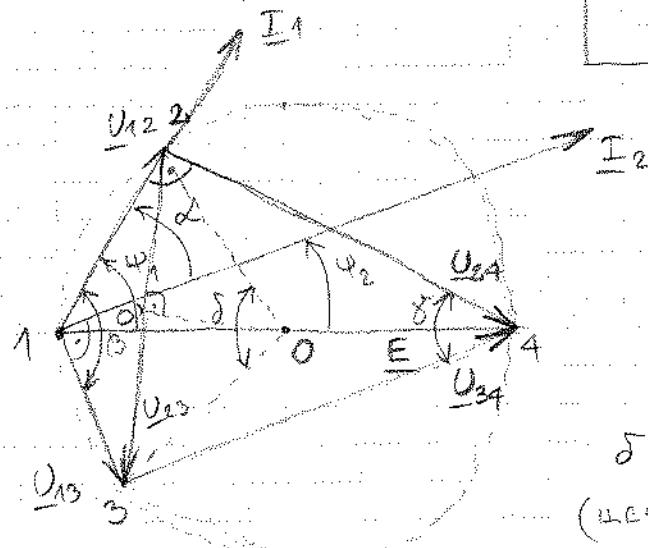
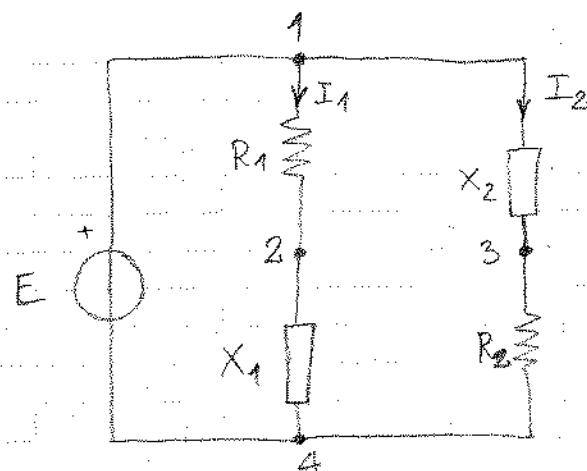
$$I = \sqrt{I_1^2 + I_2^2} = \sqrt{20^2 + 10^2} = \sqrt{400 + 100} = \sqrt{500}$$

$$I = 10\sqrt{5} A$$

$$\phi = \theta_e - \psi = -\arccos \frac{8}{10} + \arccos \frac{1}{\sqrt{1+4}} \approx 26.59^\circ$$

101. $E, \alpha = \Psi_1 - \Psi_2$

$$U_{23} = ?$$



$$U_{23} = U_{13} - U_{12}$$

$\beta = \alpha + \frac{\pi}{2}$ (периферийский угол над тетивом U_{23})

$$\gamma = \pi - \beta$$

$$\delta = 2\gamma = 2\pi - 2\beta$$

(центральный угол над тетивом U_{23})

$\Delta O'OB:$

$$\sin \frac{\delta}{2} = \frac{U_{23}}{\frac{E}{2}} = \frac{U_{23}}{E} \Rightarrow U_{23} = E \sin \frac{\delta}{2} = E \sin \gamma$$

$$U_{23} = E \sin (\pi - \beta) = E \sin \beta$$

$$\Rightarrow U_{23} = E \cdot \sin(\alpha + \frac{\pi}{2}) = E \cdot \cos \alpha$$

$$\Rightarrow \boxed{U_{23} = E \cos \alpha}$$

* допустимо: $U_{23} = E \cdot |\cos \alpha|$

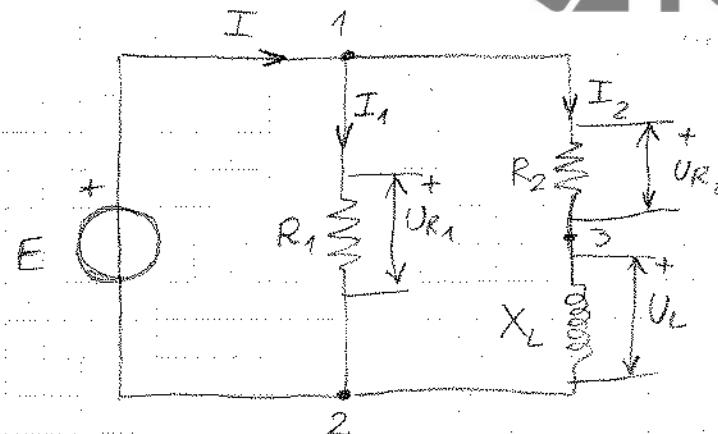
$$102. R_1 = 100 \Omega$$

$$X_L = 50 \Omega$$

$$U_L = 5 V$$

$$\angle = \theta_u - \theta_E = 60^\circ$$

$$I = ?$$



$$\frac{E}{2} = U_L$$

$$\Rightarrow \frac{U_{R1}}{2} = U_L$$

$$U_{R1} = 2 U_L = 2 \cdot 5 = 10 V$$



$$R_1 I_1 = U_{R1} \Rightarrow I_1 = \frac{U_{R1}}{R_1} = \frac{10}{100} A = \frac{1}{10} A \Rightarrow I_1 = 0.1 A$$

$$\sin \alpha = \frac{U_{R2}}{U_{R1}} \Rightarrow U_{R2} = U_{R1} \sin \alpha = 10 \cdot \sin 60^\circ = 10 \cdot \frac{\sqrt{3}}{2}$$

$$U_{R2} = 5\sqrt{3} V$$

$$I_2 X_L = U_L \Rightarrow I_2 = \frac{U_L}{X_L} = \frac{5}{50} A = \frac{1}{10} A \Rightarrow I_2 = 0.1 A$$

$$I^2 = I_1^2 + I_2^2 + 2 I_1 I_2 \cos 30^\circ$$

$$I^2 = \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^2 + 2 \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{\sqrt{3}}{2} = \frac{2}{100} + \frac{\sqrt{3}}{100} = \frac{2 + \sqrt{3}}{100}$$

$$I = \sqrt{\frac{2 + \sqrt{3}}{100}} = \frac{1}{10} \sqrt{2 + \sqrt{3}} A$$

$$I \approx 0.1932 A \Rightarrow I \approx 193 mA$$

* 103.

* 109.

3.3. КОМПЛЕКСНИ ДОМЕН

105. а) $i_1(t) = -2\sqrt{2} \sin(\omega t - \frac{\pi}{3}) A$

б) $i_2(t) = 4\sqrt{2} \cos(\omega t - \frac{3\pi}{4}) A$

в) $i_3(t) = -(\sqrt{6} \cos \omega t + 2\sqrt{6} \sin(\omega t - \frac{\pi}{6})) A$

$$\omega = 100\pi \text{ rad/s}$$

$$i(t) = I \sqrt{2} \cos(\omega t + \psi)$$

комплексни
представник

$$\rightarrow I = I e^{j\psi} = I (\cos \psi + j \sin \psi) = I' + j I''$$

а) $i_1(t) = -2\sqrt{2} \cos(\omega t - \frac{\pi}{3} - \frac{\pi}{2})$

$$i_1(t) = 2\sqrt{2} \cos(\omega t - \frac{\pi}{3} + \frac{\pi}{2}) = 2\sqrt{2} \cos(\omega t + \frac{\pi}{6}) A$$

$$\left. \begin{array}{l} I_1 = 2A \\ \psi_1 = \frac{\pi}{6} \end{array} \right\} I_1 = 2e^{j\frac{\pi}{6}} = 2(\cos \frac{\pi}{6} + j \sin \frac{\pi}{6}) A = 2 \cdot \left(\frac{\sqrt{3}}{2} + j \frac{1}{2} \right) A$$

$$I_1 = (\sqrt{3} + j) A$$

б) $i_2(t) = 4\sqrt{2} \cos(\omega t - \frac{3\pi}{4}) A$

$$\left. \begin{array}{l} I_2 = 4A \\ \psi_2 = -\frac{3\pi}{4} \end{array} \right\} I_2 = 4e^{-j\frac{3\pi}{4}} A$$

$$I_2 = 4(\cos(\frac{3\pi}{4}) - j \sin(\frac{3\pi}{4})) A$$

$$I_2 = 4 \left(-\frac{\sqrt{2}}{2} - j \frac{\sqrt{2}}{2} \right) A = (-2\sqrt{2} - j 2\sqrt{2}) A$$

$$I_2 = -2\sqrt{2}(1+j) A$$

в) $i_3(t) = -(\sqrt{6} \cos \omega t + 2\sqrt{6} \cos(\omega t - \frac{\pi}{6} - \frac{\pi}{2})) A$

$$I_3' \qquad \qquad \qquad I_3''$$

$$I_3' = \sqrt{3} e^{j0^\circ} A = \sqrt{3} A$$

$$I_3'' = 2\sqrt{3} e^{-j\frac{2\pi}{3}} = 2\sqrt{3} e^{-j\frac{2\pi}{3}} = 2\sqrt{3} \left(\cos \frac{2\pi}{3} - j \sin \frac{2\pi}{3} \right) A$$

$$I_3'' = 2\sqrt{3} \left(-\frac{1}{2} - j \frac{\sqrt{3}}{2} \right) A = (-\sqrt{3} - j \cdot 3) A$$

$$I_3' = -(\sqrt{3} + 3j) A$$

$$I_3 = - (I_3' + I_3'') = - (\sqrt{3} + (-(\sqrt{3} + j\sqrt{3}))) A$$

$$I_3 = - (\sqrt{3} - \sqrt{3} - j\sqrt{3}) A = - (-j\sqrt{3}) A = j\sqrt{3} A$$

$$\boxed{I_3 = +j\sqrt{3} A}$$

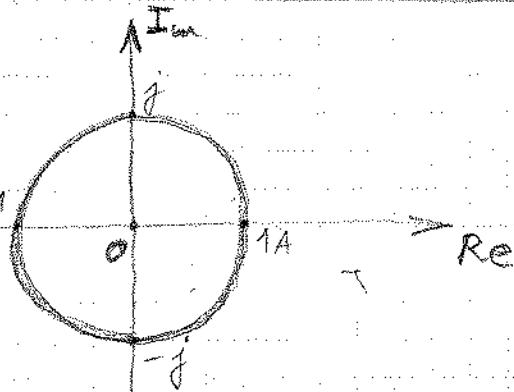
106. w

a) $I = +1A$

b) $I = -1A$

c) $I = jA$

d) $I = -jA$



a) $I = (1 + j \cdot 0) A = (\cos 0 + j \sin 0) A = 1 \cdot (\cos 0 + j \sin 0) A$

$$I = e^{j0^\circ} A \Rightarrow I = 1A \quad \varphi = 0$$

$$\boxed{i(t) = \sqrt{2} \cos(\omega t) A}$$

b) $I = -1A = (-1 + j \cdot 0) A = (\cos \pi + j \sin \pi) A$

$$I = e^{j\pi} A \Rightarrow I = 1A \quad \varphi = \pi$$

$$i(t) = \sqrt{2} \cos(\omega t + \pi)$$

$$\boxed{i(t) = -\sqrt{2} \cos(\omega t)}$$

c) $I = jA = (0 + j \cdot 1) A = (\cos \frac{\pi}{2} + j \sin \frac{\pi}{2}) A$

$$I = e^{j\frac{\pi}{2}} A \Rightarrow I = 1A \quad \varphi = \frac{\pi}{2}$$

$$i(t) = \sqrt{2} \cos(\omega t + \frac{\pi}{2}) = -\sqrt{2} \cos(\omega t - \frac{\pi}{2})$$

$$\boxed{i(t) = -\sqrt{2} \sin(\omega t)}$$

d) $I = -jA = (0 + j(-1)) A = (\cos \frac{3\pi}{2} + j \frac{3\pi}{2}) A$

$$I = e^{j\frac{3\pi}{2}} = e^{j - \frac{\pi}{2}} = e^{-j\frac{\pi}{2}} \Rightarrow I = 1A \quad \varphi = -\frac{\pi}{2}$$

$$i(t) = \sqrt{2} \cos(\omega t - \frac{\pi}{2})$$

$$\boxed{i(t) = \sqrt{2} \sin(\omega t) A}$$

107. $\omega = 400\pi \text{ rad/s}$

a) $U_1 = 50(1-j\sqrt{3}) \text{ V}$

$\delta)$ $U_2 = 25\sqrt{2}(\sqrt{3}+j)e^{-j\frac{3\pi}{2}} \text{ V}$

b) $U_3 = \left(\frac{50}{3-j\cdot 4} - \frac{24}{1-j\sqrt{3}} + 4(2+j\frac{3\sqrt{3}}{2}) \right) \text{ V}$

НАДУСАТИ ТРЕНУЩЕ ИМЕНІ ЗЧЕТЕ НАНОА:

a) $U_1 = 50(1-j\sqrt{3}) \text{ V}$

$$U_1 = |50(1-j\sqrt{3})| = |50| \cdot |1-j\sqrt{3}| = 50 \cdot \sqrt{1+3} = 50 \cdot 2 = 100 \text{ V}$$

$$\theta_1 = \arg(50(1-j\sqrt{3})) = \arg 50 + \arg(1-j\sqrt{3}) = 0 + \arctg \frac{-\sqrt{3}}{1}$$

$$\theta_1 = -\frac{\pi}{3}$$

$$u_1(t) = 100\sqrt{2} \cos(\omega t - \frac{\pi}{3}) \text{ V}$$

$$(u_1(t) = 100\sqrt{2} \cos(400\pi t - \frac{\pi}{3}) \text{ V})$$

$\delta)$ $U_2 = |25\sqrt{2}(\sqrt{3}+j)e^{-j\frac{3\pi}{2}}| = |25\sqrt{2}| \cdot |\sqrt{3}+j| \cdot |e^{-j\frac{3\pi}{2}}|$

$$U_2 = 25\sqrt{2} \cdot \sqrt{3+1} \cdot \sqrt{1} = 25\sqrt{2} \cdot 2 = 50\sqrt{2} \text{ V}$$

$$\theta_2 = \arg(25\sqrt{2} \cdot (\sqrt{3}+j)e^{-j\frac{3\pi}{2}}) = \arg(25\sqrt{2}) + \arg(\sqrt{3}+j) + \arg(e^{-j\frac{3\pi}{2}})$$

$$\theta_2 = 0 + \arctg \frac{1}{\sqrt{3}} - \frac{3\pi}{2} = \frac{\pi}{6} - \frac{3\pi}{2} = \frac{10-9\pi}{6} = -\frac{28\pi}{6} = -\frac{4\pi}{3}$$

$$\theta_2 = 0 + \arctg \frac{1}{\sqrt{3}} + \arg(0+j \cdot 1) = \frac{\pi}{6} + \frac{\pi}{2} = \frac{1+3}{6}\pi = \frac{4}{6}\pi = \frac{2}{3}\pi$$

$$\Rightarrow u_2(t) = 100 \cos(400\pi t - \frac{\pi}{3}) \text{ V}$$

b) $U_3 = \frac{50}{3-j\cdot 4} \cdot \frac{3+j\cdot 4}{3+j\cdot 4} - \frac{24}{1-j\sqrt{3}} \cdot \frac{1+j\sqrt{3}}{1+j\sqrt{3}} + 8 + j6\sqrt{3}$

$$U_3 = \frac{50}{25} (3+j\cdot 4) - \frac{24}{2} (1+j\sqrt{3}) + 8 + j6\sqrt{3}$$

$$U_3 = 8 + j \cdot 8 - 6 - j6\sqrt{3} + 8 + j6\sqrt{3} = 8(1+j)$$

$$U_3 = |8 \cdot (1+j)| = |8| \cdot |1+j| = 8 \cdot \sqrt{1+1} = 8\sqrt{2} \text{ V}$$

$$\theta_3 = \arg(8(1+j)) = \arg(8) + \arg(1+j) = 0 + \arctg \frac{1}{1} = \frac{\pi}{4}$$

$$\boxed{U_3(t) = 16 \cos(\omega_0 t + \frac{\pi}{4}) V}$$



108. $\underline{U} = -110(\sqrt{3} + j)V$

1) У комплексном домену:

$$\underline{U}_n = -\underline{U} = \underline{U} \cdot e^{\pm j\pi} = -110(\sqrt{3} + j) \cdot (-1) = \boxed{110(\sqrt{3} + j)}$$

$$= U_n e^{j\theta_n}$$

$$220 \cdot e^{j\pi} \\ 220(\cos \frac{\pi}{6} + j \sin \frac{\pi}{6})$$

2) У временному домену:

$$U(t) = U \cos(\omega t + \theta)$$

$$U = |-110(\sqrt{3} + j)| = |-110| \cdot |\sqrt{3} + j| = 110 \cdot \sqrt{3+1} = 110 \cdot 2 V$$

$$\boxed{U = 220 V}$$

$$\theta = \arg(-110(\sqrt{3} + j)) = \arg(110) + \arg(\sqrt{3} - j) = 0 + \arg \frac{1}{\sqrt{3}} - \pi$$

$$\theta = \frac{\pi}{6} - \pi \quad \theta = -\frac{5\pi}{6}$$

$$U(t) = 220\sqrt{2} \cos(\omega t - \frac{5\pi}{6})$$

$$u_n(t) = -220\sqrt{2} \cos(\omega t - \frac{5\pi}{6})$$

$$u_n(t) = 220\sqrt{2} \cos(\omega t - \frac{5\pi}{6} + \pi) = 220\sqrt{2} \cos(\omega t + \frac{7\pi}{6})$$

$$u_n(t) = 220\sqrt{2} \cos(\omega t + \frac{\pi}{6})$$

109. $\underline{U} = -110(\sqrt{3} + j)$

1) У временному домену:

$$U = 220$$

$$u(t) = 220\sqrt{2} \cos(\omega t - \frac{5\pi}{6})$$

$$\Delta t_p = \frac{T}{8} = \frac{2\pi}{8\omega} = \frac{\pi}{4\omega} \rightarrow \text{КАШЕЧКА}$$

$$t^{(1)} = t - \Delta t_p$$

$$u^{(1)}(t^{(1)}) = 220\sqrt{2} \cos(\omega(t^{(1)} + \Delta t_p) - \frac{5\pi}{6})$$

$$u^{(1)}(t^{(1)}) = 220\sqrt{2} \cos(\omega t^{(1)} + \underbrace{\omega \Delta t_p}_{\theta^{(1)}} - \frac{5\pi}{6})$$

$$u^{(1)}(t^{(1)}) = 220\sqrt{2} \cos(\omega t^{(1)} + \theta^{(1)})$$

$$\theta^{(1)} = \theta + \omega \Delta t_p = -\frac{5\pi}{6} + \frac{7}{9} = \frac{-10+3}{12}\pi = -\frac{7}{12}\pi$$

* почестна фаза се повдигана за

$$-\frac{7}{12}\pi$$

2) в комплексном домене:

$$\underline{U}^{(1)} = UV_2 e^{j\theta^{(1)}} = U\sqrt{2} e^{j\theta} \cdot e^{j\omega t_p} = U e^{j\omega t_p}$$

$$\underline{U}^{(1)} = -110(\sqrt{3} + j)(\cos \frac{\pi}{3} + j \sin \frac{\pi}{3})$$

$$\underline{U}^{(1)} = -110(\sqrt{3} + j)\left(\frac{\sqrt{3}}{2} + j\frac{\sqrt{3}}{2}\right)V$$

$$\underline{U}^{(1)} = -55\sqrt{2}(\sqrt{3} + j)(1+j)V$$

$$\underline{U}^{(1)} = -55\sqrt{2}(\sqrt{3} + j + \sqrt{3}j + (-1))V = -55\sqrt{2}(\sqrt{3}-1 + j(\sqrt{3}+1))V$$

$$\underline{U}^{(1)} = -55\sqrt{2}(\sqrt{3}-1 + j(\sqrt{3}+1))V$$

110. $\underline{U} = -110(\sqrt{3} + j)V$

$$\underline{U}^{(1)} = -U\sqrt{2} e^{j\theta^{(1)}} = U\sqrt{2} e^{j\theta} \cdot e^{j\omega t_p}$$

$$\underline{U}^{(1)} = U\sqrt{2} e^{j\theta} \cdot e^{j\pi} \cdot e^{j\frac{\pi}{3}} = U \cdot e^{j\pi} \cdot e^{j\frac{\pi}{3}}$$

$$\underline{U}^{(1)} = -110(\sqrt{3} + j) \cdot (-1) \cdot \left(\frac{\sqrt{3}}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$\underline{U}^{(1)} = 55\sqrt{2}(\sqrt{3} + j)(1+j)V = 55\sqrt{2}(\sqrt{3}-1 + j + j\sqrt{3})V$$

$$\underline{U}^{(1)} = 55\sqrt{2}(\sqrt{3}-1 + j(\sqrt{3}+1))V$$

111. $I = 20(\sqrt{3} + j)mA$

$$t^{(1)} = t + \Delta t_p \Leftrightarrow t = t^{(1)} - \Delta t_p \quad \Delta t_p = \frac{T}{4} = \frac{2\pi}{2\omega}$$

$$I^{(1)} = IV_2 e^{j\varphi^{(1)}} = IV_2 e^{j\varphi} (\psi - \omega \Delta t_p) = IV_2 e^{j\psi} \cdot e^{-j\omega \Delta t_p}$$

$$I^{(1)} = I \cdot e^{-j\frac{\pi}{2}} = 20(\sqrt{3} + j) \cdot (0 - j)A$$

$$I^{(1)} = 20(-j\sqrt{3} + 1)A \Rightarrow (I^{(1)} = 20(1 - j\sqrt{3})mA)$$

$$112. \text{ to } \rightarrow I = (\sqrt{3} + j) \text{ mA}$$

$$t_1 \rightarrow I^{(1)} = (1 + j\sqrt{3}) \text{ mA}$$

$$t_2 \rightarrow \Delta t_{p2} = t_2 - t_0 = -\frac{11}{2} \Delta t_{p1}$$

$$\Delta t_{p1} = t_1 - t_0, |\Delta t_{p1}| \leq \frac{T}{2} \rightarrow T \text{ є ПЕРИОД ПОСМОТРАНЕ СТРУДЕ}$$

$$I^{(1)} = I e^{j\omega \Delta t_{p1}} = I e^{j\psi^{(1)}} \quad \left\{ \begin{array}{l} \psi^{(1)} = \psi + \omega \Delta t_{p1} \\ \psi^{(1)} - \psi = \Delta \psi^{(1)} = \omega \Delta t_{p1} \end{array} \right.$$

$$I = I e^{j\psi} \quad \left\{ \begin{array}{l} \psi^{(1)} = \psi + \omega \Delta t_{p1} \\ \psi^{(1)} - \psi = \Delta \psi^{(1)} = \omega \Delta t_{p1} \end{array} \right.$$

$$\Psi = \arg(\sqrt{3} + j) = \arctg \frac{1}{\sqrt{3}} = \frac{\pi}{6} \Rightarrow \Delta t_{p1} = \frac{\Delta \psi^{(1)}}{\omega}$$

$$\Psi^{(1)} = \arg(1 + j\sqrt{3}) = \arctg \sqrt{3} = \frac{\pi}{3}$$

$$\psi^{(1)} - \psi = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

$$\Rightarrow \Delta \psi^{(1)} = \frac{\pi}{6}$$

$$\Delta t_{p1} = \frac{\pi}{6\omega} = \frac{\pi}{6 \cdot 2\pi} = \frac{1}{12}$$

$$\Delta t_{p2} = -\frac{11}{2} \cdot \frac{\pi}{12} = -\frac{11\pi}{24}$$

$$\psi^{(2)} - \psi^{(1)} = \Delta \psi^{(2)} = \pi + \omega \Delta t_{p2} = \pi + \frac{2\pi}{T} \left(-\frac{11}{2} \cdot \frac{\pi}{12} \right)$$

$$\psi^{(2)} - \psi^{(1)} = \pi - \frac{11}{12}\pi \Rightarrow \psi^{(2)} - \psi = \frac{5\pi}{12}$$

$$\psi^{(2)} = \frac{\pi}{12} + \psi^{(1)} = \frac{\pi}{12} + \frac{\pi}{3} = \frac{5\pi}{12}$$

$$I^{(2)} = I e^{j\Delta \psi^{(2)}} = 2 e^{j\frac{\pi}{12}} \cdot e^{j\frac{5\pi}{12}} = 2 e^{j\frac{6\pi}{12}} \text{ mA}$$

$$I^{(2)} = 2 e^{j\frac{\pi}{2}} \text{ mA} = 2 \cdot \left(\frac{\sqrt{2}}{2} + j \frac{\sqrt{2}}{2} \right) = \sqrt{2} (j+1) \text{ mA}$$

$$(I^{(2)}) = \sqrt{2} (1+j) \text{ mA}$$

113.

114. $i(t) = 100 \cos(\omega t)$ A $\omega = 10^3 \text{ s}^{-1}$

a) $\frac{di(t)}{dt} = \frac{d}{dt}(100 \cos(\omega t)) = 100(-\omega \sin(\omega t))$ A

$$\frac{di}{dt} = -100\omega \cos\left(\omega t - \frac{\pi}{2}\right) = 10^2 \cdot 10^3 \cos\left(\omega t + \frac{\pi}{2}\right)$$

$$\left. \frac{di}{dt} = 10^5 \cos\left(\omega t + \frac{\pi}{2}\right) \text{ As}^{-1} \right\}$$

b) $I = \frac{100}{\sqrt{2}} = 50\sqrt{2}$ { } $I = 50\sqrt{2} \cdot e^{j0^\circ}$ A = $50\sqrt{2}$ A
 $\psi = 0$

$$j\omega I = j \cdot 10^3 \cdot 50\sqrt{2} \text{ A} \Rightarrow j\omega I = j5\sqrt{2} \cdot 10^4 \text{ As}^{-1}$$

115. $U = (-1 - j\sqrt{3})V$ $\omega = 10^3 \text{ s}^{-1}$

a) $U = |-1 - j\sqrt{3}| = \sqrt{1+3} = \sqrt{4} = 2$

$$\omega = 10^3 \text{ s}^{-1}$$

$$\theta = \arg(-1 - j\sqrt{3}) = \arctg \frac{-\sqrt{3}}{1} - \pi = \frac{2\pi}{3} - \pi = -\frac{2\pi}{3}$$

$$u(t) = 2\sqrt{2} \cos\left(10^3 t - \frac{2\pi}{3}\right)$$

$$\int u(t) dt = \int 2\sqrt{2} \cos\left(\omega t - \frac{2\pi}{3}\right) dt$$

$$= 2\sqrt{2} \cdot \frac{1}{\omega} \sin\left(\omega t - \frac{2\pi}{3}\right) = \frac{2\sqrt{2}}{\omega} \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$\int u(t) dt = 2\sqrt{2} \cdot 10^{-3} \cos\left(\omega t - \frac{2\pi}{3} - \frac{\pi}{2}\right)$$

$$\int u(t) dt = 2\sqrt{2} \cdot 10^{-3} \cdot \cos\left(\omega t - \frac{4\pi}{6}\right)$$

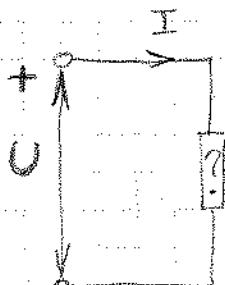
$$\int u(t) dt = 2\sqrt{2} \cdot 10^{-3} \cdot \cos\left(\omega t - \frac{7\pi}{6}\right)$$

$$\left. \int u(t) dt = 2\sqrt{2} \cdot 10^{-3} \cdot \cos\left(\omega t + \frac{5\pi}{6}\right) V_s \right\}$$

$$d) \frac{U}{j\omega} = \frac{-1-j\sqrt{3}}{j \cdot 10^3} \cdot \frac{j}{j} = \frac{-j+\sqrt{3}}{-10^3} = \frac{1}{10^3} (-\sqrt{3}+j)$$

$$\left\{ \frac{U}{j\omega} = 10^{-3} (-\sqrt{3}+j) \text{ Vs} \right.$$

116.



ВРЕМЕНСКИ ДОМЕН

$$i(t) = I\sqrt{2} \cos(\omega t + \Phi)$$

$$u(t) = U\sqrt{2} \cos(\omega t + \theta)$$

$$Z = \frac{U}{I} = \frac{1}{2} \rightarrow \text{ИМПЕДАНСА } (Z > 0)$$

$$\phi = \theta - \Phi \rightarrow \text{РАЗЛИКА } \neq \text{ФАЗА}$$

$$Y = \frac{I}{U} = \frac{1}{2} \rightarrow \text{АДМИТАНСА } (Y > 0)$$

$$\gamma = \Phi - \phi \rightarrow \text{РАВНА } \text{РАЗЛИКА}$$

КОМПЛЕКСНИ ДОМЕН

$$U = U \cdot e^{j\Phi}$$

$$I = I \cdot e^{j\Phi}$$

$$Z = Z I \rightarrow Z = \text{КОМПЛЕКСНА ИМПЕДАНСА ПРИЈЕМНИКА}$$

$$Y = \frac{1}{Z} = \frac{I}{U} \rightarrow Y = \text{КОМПЛЕКСНА АДМИТАНСА}$$

$$Z = Z e^{j\Phi}, |Z| = Z \wedge \arg(Z) = \Phi$$

$$Z = R + jX$$

$$R = \operatorname{Re}(Z) = Z \cos \Phi \quad \text{РЕЗИСТАНСА } (\text{АКТИВНА отпорност})$$

$$X = \operatorname{Im}(Z) = Z \sin \Phi \quad \text{РЕАКТАНСА } (\text{РЕАКТИВНА отпорност})$$

$$Z = \sqrt{R^2 + X^2} \wedge \Phi = \arctan \frac{X}{R}$$

$$Y = Y e^{j\Phi} = G + jB, Y = |\Sigma| \wedge \Psi = \arg(Z)$$

$$G = \operatorname{Re}(Y) = Y \cos \Psi \quad \text{КОНДУКТАНСА } (\text{АКТИВНА проводност})$$

$$B = \operatorname{Im}(Y) = Y \sin \Psi \quad \text{СУЩЕЛТАНСА } (\text{РЕАКТИВНА проводност})$$

РЕДАЦИЈЕ:

$$R = \frac{G}{G^2 + B^2} = \frac{G}{Y^2} = GZ^2$$

$$X = \frac{-B}{G^2 + B^2} = \frac{-B}{Y^2} = -BZ^2$$

$$G = \frac{R}{R^2 + X^2} = \frac{R}{Z^2} = RY^2$$

$$B = \frac{-X}{R^2 + X^2} = \frac{-X}{Z^2} = -XY^2$$

$$ZY = 1$$

$$\phi + \psi = 0$$

117. $i(t) = -\sqrt{2} \sin \omega t \text{ A}$

$$\omega = 10^3 \text{ s}^{-1}$$

$$U = 5 \text{ V}$$

$$\phi = \theta - \psi = -\frac{\pi}{4}$$

$$Z = ? \quad Y = ?$$

$$i(t) = -1 \cdot \sqrt{2} \cos(\omega t - \frac{\pi}{2}) = 1 \cdot \sqrt{2} \cos(\omega t + \frac{\pi}{2})$$

$$I = 1 \quad \psi = \frac{\pi}{2}$$

$$I = I e^{j\frac{\pi}{2}} = I j \rightarrow I = j A$$

$$\theta = \phi - \psi = -\frac{\pi}{4} \Rightarrow \theta = -\frac{\pi}{4} + \psi = -\frac{\pi}{4} + \frac{\pi}{2} = \frac{-1+2}{4}\pi$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

$$U = U e^{j\theta} = U \cdot e^{j\frac{\pi}{4}} = U \left(\frac{\sqrt{2}}{2} + j \frac{\sqrt{2}}{2} \right) = \boxed{5 \left(\frac{\sqrt{2}}{2} + j \frac{\sqrt{2}}{2} \right) \text{ V}}$$

$$Z = \frac{U}{I} = \frac{\frac{5\sqrt{2}}{2} + j \frac{5\sqrt{2}}{2}}{j} \cdot j = \frac{\frac{5\sqrt{2}}{2} - \frac{5\sqrt{2}}{2}}{-1} = \frac{5\sqrt{2}}{2} - j \frac{5\sqrt{2}}{2}$$

$$Z = \frac{5\sqrt{2}}{2} (1-j) \Omega = 2.5\sqrt{2} (1-j) \Omega$$

$$Y = \frac{1}{Z} = \frac{I}{U} = \frac{j}{5\sqrt{2} - j5\sqrt{2}} \cdot \frac{\frac{5\sqrt{2}}{2} - j\frac{5\sqrt{2}}{2}}{\frac{5\sqrt{2}}{2} - j\frac{5\sqrt{2}}{2}} = \frac{j \frac{5\sqrt{2}}{2} + j \frac{5\sqrt{2}}{2}}{25 - 25} = \frac{5\sqrt{2}}{50} + j \frac{5\sqrt{2}}{50}$$

$$Y = 0.1\sqrt{2}(1+j) S$$

$$118. \quad I = (21 + j72) \mu A$$

$$U = (-9 + j12) V$$

а) КАРАКТЕРИСТИКЕ ОТПОРНОСТИ И ЕКВИВАЛЕНТНА ШЕМА

б) КАРАКТЕРИСТИКЕ ПРОВОДНОСТИ И ЕКВИВАЛЕНТНА ШЕМА.

$$\omega = 10^3 s^{-1}$$

$$a) \quad Z = \frac{U}{I} = \frac{-9 + j \cdot 12}{21 + j \cdot 72} \cdot \frac{21 - j \cdot 72}{21 - j \cdot 72} = \frac{-189 + 864 + j(648 + 252)}{441 + 5184}$$

$$Z = \frac{675 + j(900)}{5625} \quad k\Omega = \left(\frac{3}{25} + j\frac{9}{25}\right) \cdot 10^3 \Omega$$

$$Z = (120 + j160) \Omega$$

$$R = \operatorname{Re}(Z) = 120 \Omega$$

$$X = \operatorname{Im}(Z) = 160 \Omega$$

$$Z = \operatorname{mod}(Z) = \sqrt{120^2 + 160^2} = \sqrt{100 \cdot (194 + 256)} = \sqrt{100 \cdot 450}$$

$$Z = 200 \Omega$$

$$\phi = \arg(Z) = \arctg \frac{160}{120} = \arctg \frac{4}{3} = 0.927 \approx 0.93$$

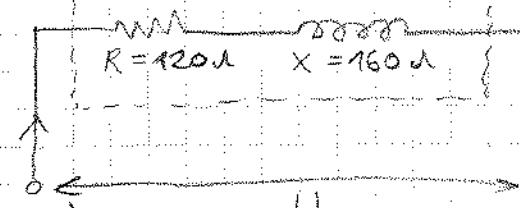
$X > 0 \Rightarrow$ приемник је претежно индуктиван

→ РЕДНА ВЕЗА ОТПОРНИКА И КАПЕМА

$$R = 120 \Omega$$

$$L = \frac{X}{\omega} = \frac{160}{10^3} H = 160 \mu H$$

$$Z = R + j\omega L$$



$$\underline{Y} = \frac{\underline{I}}{\underline{U}} = \frac{21 + j72}{-3 + j12} \cdot \frac{-3 - j12}{-3 - j12} = \frac{-189 + 864 - j(678 + 252)}{81 + 144}$$

$$\underline{Y} = \frac{675 - j(900)}{225} = (3 - j4) \text{ mS}$$

$$G = \operatorname{Re}(\underline{Y}) = 3 \text{ mS}$$

$$B = \operatorname{Im}(\underline{Y}) = -4 \text{ mS}$$

$$Y = \operatorname{mod}(\underline{Y}) = \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25}$$

$$Y = 5 \text{ mS}$$

$$\vartheta = \arg(3 - j4) = \arctg \frac{-4}{3} = -0.927 \approx -0.93 \text{ rad}$$

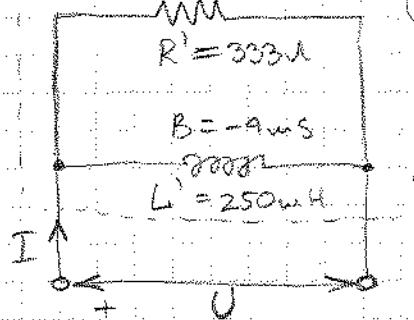
$$R' = \frac{1}{G} = \frac{1}{\frac{3}{10^{-3}}} = 333 \Omega$$

$$L' = \frac{1}{-jB} = \frac{1}{+10^{-3} \cdot 4 \cdot 10^{-3}} = 0.25 \text{ H} = 250 \mu\text{H}$$

ПРИЈЕМНИК

$$\left\{ \underline{Y} = \frac{1}{R'} + \frac{1}{j\omega L'} \right\}$$

$$G = 3 \text{ mS}$$

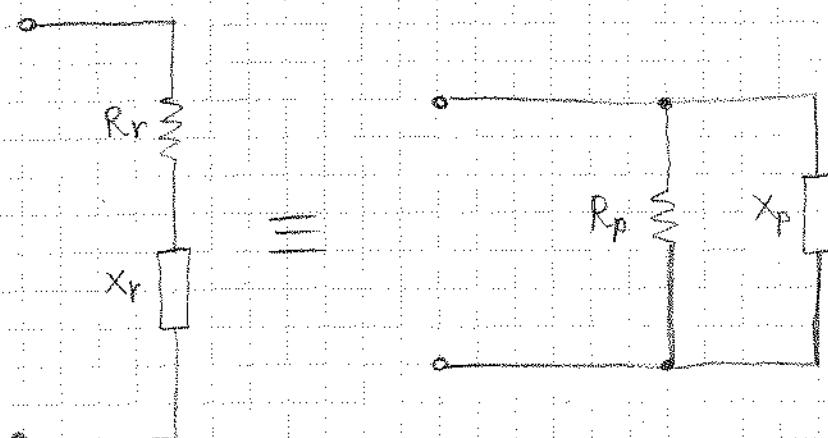


119. R_r, X_r

$$R_p = ? \quad X_p = ?$$

a) $R_r \ll |X_r|$

b) $R_r \gg |X_r|$



ВАЖЕ РЕЛАЦИЈЕ:

$$\frac{1}{R_p} = \frac{R_r}{R_r^2 + X_r^2} \Rightarrow R_p = \frac{R_r^2 + X_r^2}{R_r}$$

$$\frac{1}{X_p} = \frac{X_r}{R_r^2 + X_r^2} \Rightarrow X_p = \frac{R_r^2 + X_r^2}{X_r}$$

$$X_p \neq X_r$$

a) $R_r \ll |X_r| \Rightarrow R_p \approx \frac{X_r^2}{R_r} \wedge X_p \approx X_r$

d) $R_r \gg |X_r| \Rightarrow R_p \approx R_r \wedge X_p \approx \frac{R_r^2}{X_r}$

120. $I_g = 2A$

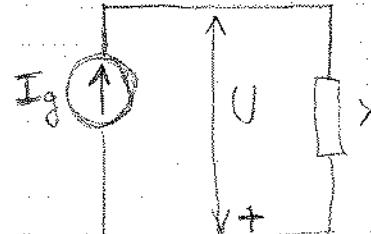
$$\omega = 10^3 s^{-1}$$

$$Y_g = -\frac{x}{2}$$

$$Y = 10mS$$

$$R = 60\Omega$$

ПРЕДЕЛНО КАРАНТ



$$U = ?$$

$$U = -Z I_g$$

$$I_g = I_g e^{j\varphi_g} = I_g e^{-j\frac{\pi}{2}} = -2jA$$

$$Z = \frac{1}{Y} = \frac{1}{10} \text{ k}\Omega = 100\Omega$$

$$Z^2 = R^2 + X^2 \Rightarrow X^2 = Z^2 - R^2 = 10^4 - 60^2$$

$$X = \pm \sqrt{100(100 - 36)} = \pm \sqrt{100 \cdot 64} = \pm 10 \cdot 8 = \pm 80\Omega$$

$$\Rightarrow X = -80\Omega$$

$$Z = (60 - j \cdot 80)\Omega$$

$$U = -Z \cdot I_g = -(60 - j \cdot 80) \cdot (-2j) = 120j - 160 (A)^2$$

$$U = (160 + j120)V$$

121. $U = 100 \text{ V}$

$$u(t_1)_{\max} \rightarrow i(t_1) = 10 \text{ mA} \text{ и отада}$$

$$i(t_2)_{\max} \rightarrow u(t_2) = 100 \text{ V}$$

$$Z = ?$$

$$\phi = 0 \text{ из условия!}$$

$$i(t) = I\sqrt{2} \cos \omega t = \frac{U}{Z}\sqrt{2} \cos \omega t$$

$$u(t) = U\sqrt{2} \cos(\omega t + \phi)$$

$$i(t_2)_{\max} \rightarrow u(t_2) = U\sqrt{2} \cos \phi = U = 100 \text{ V}$$

$$\Rightarrow \sqrt{2} \cos \phi = 1$$

$$\Rightarrow \cos \phi = \frac{\sqrt{2}}{2} \Rightarrow \boxed{\phi = \frac{\pi}{4}}$$

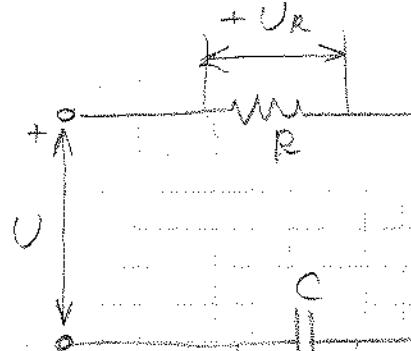
$$u(t_1)_{\max} \rightarrow i(t_1) =$$

122. $R = 20 \Omega$

$$U_R = U_C = 160 \text{ V}$$

a) $U = ?$

d) $\theta_U - \theta_{U_C} = ?$



$I \rightarrow$ комплексна таука стряже

$$U_R = Z_R I = R I$$

$$U_C = Z_C I = \frac{1}{j\omega C} I$$

$$U_R = |U_R| = |R I| = R |I| = R I \Rightarrow I = \frac{U_R}{R} = \frac{160}{20} = 8 \text{ A}$$

$$U_C = |U_C| = \left| \frac{I}{j\omega C} \right| = \left| \frac{1}{j\omega C} \right| |I| = \frac{|I|}{\omega C}$$

$$\Rightarrow U_R = U_C \Leftrightarrow R I = \frac{I}{\omega C} \Leftrightarrow R = \frac{1}{\omega C} = 20$$

$$Z = Z_R + Z_C = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C} = R(1 - j)$$

$$Z = 20(1 - j) \Omega = (20 - j20) \Omega$$

$$U = Z \cdot I \Rightarrow U = |Z I| = Z I = \sqrt{2 \cdot 20^2} \cdot 8 = 20\sqrt{2} \text{ V}$$

$$\{ U = 160\sqrt{2} \text{ V} \approx 226 \text{ V} \}$$

d) $U_C = U_C e^{j\theta_C}$

$$U = U e^{j\theta}$$

$$U_C = \frac{Z_C}{Z_C + Z_R} U = \frac{Z_C}{Z} U$$

$$\frac{U}{U_C} = \frac{U e^{j\theta}}{U_C e^{j\theta_C}} = \frac{U}{U_C} e^{j(\theta - \theta_C)}$$

$$\frac{U}{U_C} = \frac{Z I}{Z_C Z} = \frac{Z}{Z_C} = \frac{20 - j20}{-j20} = \frac{-j400 - 900}{-400} = j + 1$$

$$\Rightarrow \frac{U}{U_C} e^{j(\theta - \theta_C)} = 1 + j \Rightarrow \arg(1 + j) = \alpha = \theta - \theta_C$$

$$\Rightarrow [\alpha = \arctg \frac{1}{4} = \frac{\pi}{4}] \Rightarrow [\theta - \theta_C = \frac{\pi}{4}]$$

$$U, \theta = \text{const}$$

123. $\dot{u}(t) = 220\sqrt{2} \cdot \cos(1000\pi t [s] - \frac{\pi}{4}) V$

$$i(t) = 11 \cos 1000\pi t [s] A$$

$$i(t) = ? \cdot 1000\pi \rightarrow 2000\pi$$

$$Z = \frac{U}{I} = \frac{U e^{j\theta}}{I e^{j\varphi}} = \frac{220 e^{-j\frac{\pi}{4}}}{\frac{14}{\sqrt{2}} e^{j0.1}}$$

$$Z = 20\sqrt{2} \cdot \left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) = 20(1-j) \Omega$$

$$\Rightarrow R = 20 \Omega \quad X = +20\Omega = +\frac{1}{\omega C} \Rightarrow \frac{1}{\omega C} = 20 \Omega$$

$$\Rightarrow C = \frac{1}{\omega \cdot 20} F = \frac{1}{1000\pi \cdot 20} F \approx 16 \mu F$$

$$\omega_1 = 2 \cdot 1000\pi = 2000\pi s^{-1}$$

$$R_1 = R = 20 \Omega$$

$$X_1 = -\frac{1}{\omega C} = -\frac{1}{2\omega C} = -10 \Omega$$

$$\Rightarrow Z_1 = (20 - j10) \Omega$$

$$Z_1 = 10(2-j) \Omega$$

$$\phi_1 = \arg(Z_1) = \arctg \frac{X_1}{R_1} = \arctg \frac{-10}{20} = -\arctg \frac{1}{2} \approx -0.46 \text{ rad}$$

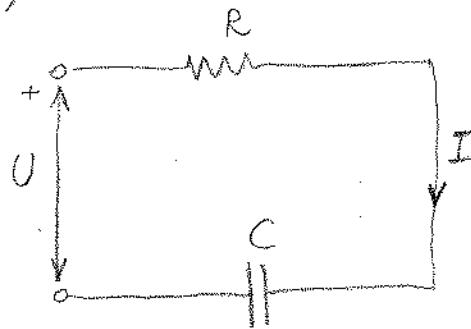
$$(\phi_1 \approx -0.46 \text{ rad})$$

$$I_1 = \frac{U}{Z_1} = \frac{220}{\sqrt{20^2 + 10^2}} = \frac{220}{\sqrt{500}} = \frac{22\sqrt{2}}{5} = 9.9\sqrt{2} A$$

$$\Psi_1 = \theta - \phi_1 = -\frac{\pi}{4} + 0.46 \text{ rad} \approx -0.102 \text{ rad} \approx -18.4^\circ$$

$$i_1(t) = I_1\sqrt{2} \cos(\omega_1 t + \Psi_1)$$

$$(i_1(t) = 9.9\sqrt{10} \cos(2000\pi t [s] - 0.102) A)$$



124. РЕДНА ВЕЗА ДВА ПРИЈЕМНИКА

I пријемник претежно индуктиван

$$G_1 = 20 \text{ mS}$$

II пријемник претежно капацитиван

$$R_2 = 20 \Omega, X_2 = -40 \Omega$$

$$R_1 = ?, X_1 = ?, \phi = \theta - \Psi = -\frac{\pi}{4}$$

$$R_1 = \frac{G_1}{G_1^2 + B_1^2}$$

$$X_1 = \frac{-B_1}{G_1^2 + B_1^2}$$

$$R = R_1 + R_2 = \frac{G_1}{G_1^2 + B_1^2} + R_2 = \frac{G_1 + R_2 (G_1^2 + B_1^2)}{G_1^2 + B_1^2}$$

$$X = X_1 + X_2 = \frac{-B_1}{G_1^2 + B_1^2} + X_2 = \frac{-B_1 + X_2 (G_1^2 + B_1^2)}{G_1^2 + B_1^2}$$

$$\phi = \theta - \Psi = -\frac{\pi}{4} = \arctan(-1) = \arctan \frac{X}{R} \Leftrightarrow \boxed{\frac{X}{R} = -1}$$

$$\Rightarrow \frac{-B_1 + X_2 (G_1^2 + B_1^2)}{G_1^2 + B_1^2} = \frac{-B_1 + X_2 (G_1^2 + B_1^2)}{G_1 + R_2 (G_1^2 + B_1^2)} = -1$$

$$-B_1 + X_2 (G_1^2 + B_1^2) = -G_1 - R_2 (G_1^2 + B_1^2)$$

$$G_1 + R_2 G_1^2 + R_2 B_1^2 - B_1 + X_2 G_1^2 + X_2 B_1^2 = 0$$

$$G_1 + R_2 G_1^2 + X_2 G_1^2 = B_1 - B_1^2 (X_2 + R_2)$$

$$G_1 + G_1^2 (R_2 + X_2) = -B_1^2 (R_2 + X_2) + B_1$$

$$B_1^2 (R_2 + X_2) - B_1 + G_1 (1 + G_1 (R_2 + X_2)) = 0$$

$$B_1^2 (-20) - B_1 \cdot 1 + 20 \cdot 10^{-3} (1 + 20 \cdot 10^{-3} (-20)) = 0$$

$$-20 B_1^2 + B_1 + 20 \cdot 10^{-3} (1 - 0.4) = 0$$

$$20 B_1^2 + B_1 - 12 \cdot 10^{-3} = 0$$

$$B_{1,1,2} = \frac{-1 \pm \sqrt{1 + 42 \cdot 10^{-3} \cdot 20 \cdot 4}}{40} = \frac{1}{40} \pm \frac{1}{40} \sqrt{1 + 960 \cdot 10^{-3}}$$

$$\frac{7}{200} - \frac{5}{200} = \frac{2}{200} = \frac{1}{100}$$

$$B_{1,1,2} = -\frac{1}{40} \pm \frac{1}{40} \sqrt{1.96} = -\frac{1}{40} \pm \frac{1}{40} \cdot \frac{2}{5}$$

$$\frac{7+5}{200} = \frac{12}{200} = -\frac{1}{100}$$

$$B_1^{(1)} = 10 \text{ mS} \quad B_1^{(2)} = -60 \text{ mS}$$

I - правденик је претежно индуктиван; $X_1 > 0$

$$\Rightarrow X_1 = \frac{-B_1}{G_1^2 + B_1^2} > 0 \Rightarrow B_1 = -60 \text{ mS}$$

$$X_1 = \frac{60 \cdot 10^{-3}}{400 \cdot 10^{-6} + 3600 \cdot 10^{-6}} = \frac{60}{4000 \cdot 10^{-6}} = \frac{60}{4000} \text{ n} = \frac{30}{200} \text{ n}$$

$$X_1 = 15 \text{ n}$$

$$R_1 = \frac{G_1}{G_1^2 + B_1^2} = \frac{20 \cdot 10^{-3}}{4000 \cdot 10^{-6}} = \frac{20}{4000} \text{ n} \Rightarrow R_1 = 5 \text{ n}$$

125. $u = 10\sqrt{2} \cos(\omega t + \frac{\pi}{2}) \text{ V}$

$$e_1 = 10\sqrt{2} \cos \omega t \text{ V}$$

$$e_2 = 20\sqrt{2} \cos(\omega t + \frac{\pi}{2}) \text{ V}$$

$$R = 100 \text{ n}$$

$$X = 300 \text{ n}$$

$$I = ?$$

$$I = \frac{E_1 + U - E_2}{Z_R + Z_X} = \frac{U + E_1 - E_2}{Z}$$

$$U = U e^{j\theta} = 10 e^{j\frac{\pi}{2}} = j10 \text{ V}$$

$$E_1 = E_1 e^{j\theta} = 10 e^{j0} = 10 \text{ V}$$

$$E_2 = E_2 e^{j\theta} = 20 e^{j\frac{\pi}{2}} = 20 j = j20 \text{ V}$$

$$Z = R + jX = (100 + j300) \text{ n} = 100 \cdot (1 + j3) \text{ n}$$

$$I = \frac{j10 + 10 - j20}{100(1 + j3)} = \frac{(10 - j10)(1 - j3)}{100(1^2 + 3^2)} = \frac{1 - 3 - j(1+3)}{100 \cdot (1+3)}$$

$$I = \frac{-2 - j4}{100} = (-0.02 - j0.04) \text{ A} = -(20 + j40) \text{ mA}$$

$$I = |I| = \sqrt{20^2 + 40^2} = \sqrt{20^2(1+4)} = 20\sqrt{5} \text{ mA}$$

$$I = 20\sqrt{5} \text{ mA}$$

$$126. R_2 = 40 \Omega$$

$$X_2 = 30 \Omega$$

$$U_1 = 20V$$

$$U_2 = 10V$$

$$\angle = \theta_2 - \theta_1 = \frac{\pi}{2}$$

$$U_{AB} = ?$$

• УЗМЕМО ПРОИЗВОДСТВО АЗУ САМО ЈЕДНЕ ПРОСТО ОДНОДИЧНЕ

ВЕЛИЧИНЕ: $\theta_1 = 0$

$$\Rightarrow \theta_2 - \theta_1 = \frac{\pi}{2} \Rightarrow \theta_2 = \frac{\pi}{2}$$

$$U_1 = U_1 e^{j\theta_1} = 20 \cdot e^{j0^\circ} V = 20V$$

$$U_2 = U_2 e^{j\theta_2} = 10 \cdot e^{j90^\circ} V = j10V$$

$$I = \frac{U_2 - U_1}{Z_2 + jX_2} = \frac{j10}{40 + j30} = \frac{40 - j30}{40^2 + 30^2} = \frac{j400 + 300}{1600 + 900}$$

$$I = \frac{300 + j400}{2500} = \left(\frac{3}{25} + j\frac{4}{25} \right) A = (0.12 + j0.16) A$$

$$I = (120 + j160) mA = 40 (3 + j4) mA$$

$$\frac{U_2}{U_1} = \frac{U_2 \cdot e^{j\theta_2}}{U_1 \cdot e^{j\theta_1}} = \frac{U_2}{U_1} \cdot e^{j(\theta_2 - \theta_1)} = \frac{U_2}{U_1} e^{j\frac{\pi}{2}} = \frac{10}{20} e^{j\frac{\pi}{2}} = 0.5j$$

$$\frac{U_2}{U_1} = \frac{Z_2 \cdot I}{Z_1 \cdot I} = \frac{Z_2}{Z_1} = \frac{R_2 + jX_2}{R_1 + jX_1}$$

$$\Rightarrow \frac{40 + j30}{R_1 + jX_1} = 0.5j \Rightarrow 40 + j30 = \frac{R_1}{2} j - \frac{X_1}{2}$$

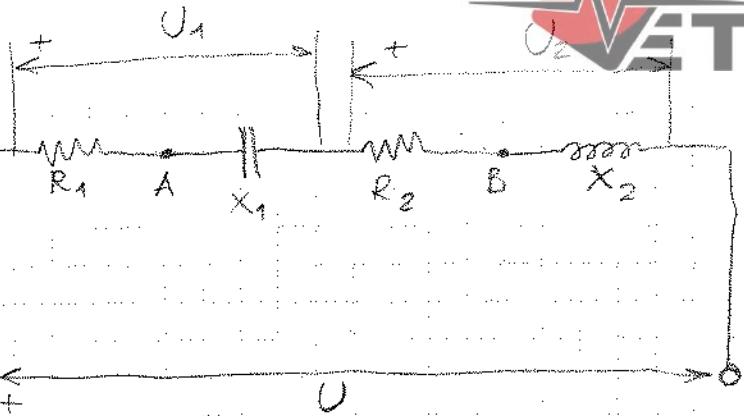
$$\Rightarrow X_1 = -2 \cdot 40 = -80 \Omega$$

$$\Rightarrow R_1 = 2 \cdot 30 = 60 \Omega$$

$$U_{AB} = (R_2 + jX_1) \cdot I = (40 - 80j) \cdot 40 (3 + 4j) \cdot 10^{-3} V$$

$$U_{AB} = (4 \cdot 8 + 12 \cdot 8) + j(-9.6 + 6 \cdot 4) = (17.6 - j3.2) V$$

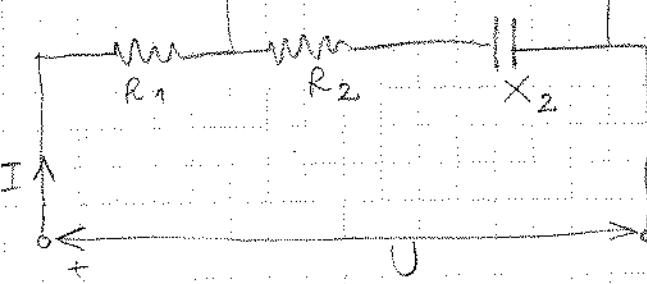
$$U_{AB} = |17.6 - j3.2| = \sqrt{17.6^2 + 3.2^2} \approx 17.9 V$$



$$127. R_2 = 10 \Omega$$

$$X_2 = -30 \Omega$$

$$R_1 = ? \quad d = \theta_2 - \theta = -\frac{\pi}{6}$$



$$\arg\left(\frac{U_2}{U}\right) = -\frac{\pi}{6}$$

$$\frac{U_2}{U} = \frac{R_2 + jX_2}{R_1 + R_2 + jX_2} \Rightarrow \frac{U_2}{U} = \frac{R_2 + jX_2}{R_1 + R_2 + jX_2}$$

$$\arg\left(\frac{U_2}{U}\right) = \arg(R_2 + jX_2) - \arg(R_1 + R_2 + jX_2) = -\frac{\pi}{6}$$

$$\Rightarrow \arctg \frac{X_2}{R_2} - \arctg \frac{X_2}{R_1 + R_2} = -\frac{\pi}{6}$$

$$\arctg \frac{-30}{10} - \arctg \frac{-30}{R_1 + 10} = -\frac{\pi}{6}$$

$$\Rightarrow \arctg \frac{-30}{R_1 + 10} = \arctg(-3) + \frac{\pi}{6} \quad / \text{tg}$$

$$\operatorname{tg}(\arctg \frac{-30}{R_1 + 10}) = \operatorname{tg}(\arctg(-3) + \frac{\pi}{6})$$

$$\operatorname{tg}(x + \beta) = \frac{\operatorname{tg}x + \operatorname{tg}\beta}{1 - \operatorname{tg}x \operatorname{tg}\beta}$$

$$\frac{-30}{R_1 + 10} = \frac{-3 + \frac{1}{\sqrt{3}}}{1 - (\sqrt{3})(\frac{1}{\sqrt{3}})} = \frac{\frac{1}{\sqrt{3}} - 3}{1 - \frac{3}{\sqrt{3}}} = \frac{1 - 3\sqrt{3}}{\sqrt{3} - 3} = \frac{1 - 3\sqrt{3}}{\sqrt{3} + 3} \cdot \frac{\sqrt{3} - 3}{\sqrt{3} - 3}$$

$$\frac{-30}{R_1 + 10} = \frac{10\sqrt{3} - 12}{4\sqrt{3}} \Rightarrow 180 = R_1(10\sqrt{3} - 12) + 100\sqrt{3} - 120$$

$$R_1(10\sqrt{3} - 12) = -100\sqrt{3} + 300$$

$$\Rightarrow R_1 = \frac{300 - 100\sqrt{3}}{10\sqrt{3} - 12} \cdot \frac{10\sqrt{3} + 12}{10\sqrt{3} + 12} = \frac{\sqrt{3} \cdot 1800 + 600}{300 - 144}$$

$$R_1 = \frac{600 + \sqrt{3} \cdot 1800}{256}$$

$$R_1 \approx 23.83 \Omega$$

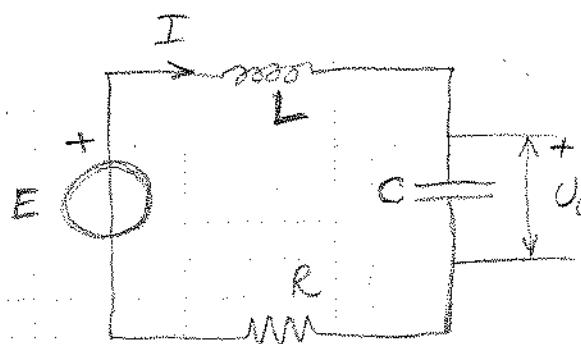
128. $L = 1 \mu H$

$$C = 100 \text{ pF}$$

$$R = 2 \Omega$$

$$f = 15.915 \text{ MHz}$$

$$U_{\max} = 1 \text{ kV}$$



$$E_{\max} = ?$$

$$\omega = 2\pi f = 31.83\pi \cdot 10^6$$

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + (wL - \frac{1}{wC})^2}}$$

$$\left| wL - \frac{1}{wC} \right| = \left| 10^2 \cdot 10^{-8} - \frac{1}{10^2 \cdot 10^{-12}} \right| \approx |10^2 - 10^{-2}| \approx R$$

\Rightarrow когда же практически \approx резонанс, и тут!

$$\Rightarrow I \approx \frac{E}{R}$$

$$U_C = Z_C I = \frac{I}{wC} \approx \frac{E}{wC} = \frac{E}{wCR}$$

$$\Rightarrow E = U_C wCR \quad U_{\max} = U_C \sqrt{2} \Rightarrow E = \frac{U_{\max}}{\sqrt{2}}$$

$$E < \frac{U_{\max} wCR}{\sqrt{2}} \approx \frac{10^3}{\sqrt{2}} 10^2 10^{-10} \cdot 2 = 10^{11-10} \frac{\text{В}}{\sqrt{2}}$$

$$E < 10\sqrt{5} \approx 14.14 \text{ V}$$

$$Q = \frac{1}{wCR} = \frac{wL}{R} \rightarrow \text{ФАКТОР ДОБРОТЕ ОЦИСЛАТОРНОГО КОЛЛА}$$

II метод: (комплексный расчет)

$$U_C = \frac{Z_C}{Z_R + Z_L + Z_C} E = \frac{\frac{1}{jwC}}{R + jwL + \frac{1}{jwC}} E$$

$$U_C \approx \frac{1}{jwC} E = \frac{E}{jwCR}$$

$$\frac{U_C}{E} = \left| \frac{1}{jwCR} \right| = \frac{1}{wCR}$$

$$E < \frac{wCR U_{\max}}{\sqrt{2}} \approx 14.14 \text{ V}$$

$$123. L = 1 \mu H$$

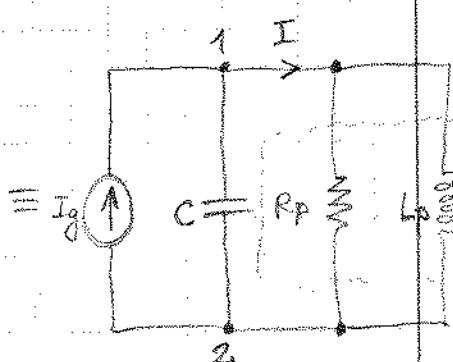
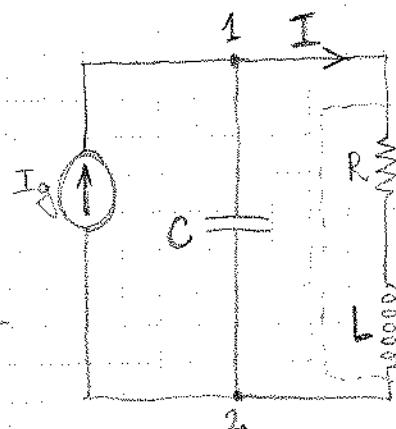
$$R = 2 \Omega$$

$$C = 100 \text{ pF}$$

$$f = 15,915 \text{ MHz}$$

$$I_{Lmax} = 1 \text{ A}$$

$$I_{gmax} = ?$$



$$R = 2 \Omega$$

$$X = \omega L \approx 2\pi f \cdot L \approx 10^8 \cdot 10^{-6} \approx 10^2 \Omega$$

$$G = \frac{1}{R_p} = \frac{R}{R^2 + (\omega L)^2} = \frac{R}{Z^2} \approx \frac{2}{10^4} = 200 \mu S = \frac{R}{(\omega L)^2} \quad (\omega L \gg R)$$

$$B = -\frac{1}{\omega L_p} = -\frac{\omega L}{R^2 + (\omega L)^2} = -\frac{\omega L}{Z^2} \approx -\frac{100}{10^4} = -10 \mu S = -\frac{1}{\omega L}$$

$$Z = \sqrt{R^2 + (\omega L)^2} \approx \omega L = 100 \Omega$$

$$\Rightarrow R_p \approx \frac{(\omega L)^2}{R} = Q, \omega L = Q^2 R = \frac{10^4}{2} = 5 k\Omega$$

$$Q_L = \frac{\omega L}{R} = \frac{100}{2} = 50 \rightarrow \text{ФАКТОР ДОБРОТЕ КАЛЕМА}$$

$$L_p \approx L = 1 \mu H$$

ЕКВИВАЛЕНТНА КОНДУКТАНСА ПАРАДОЖНЕ ВРЕМЕНО:

$$G_e = \frac{1}{R_p}$$

ЕКВИВАЛЕНТНА СЧЛЕНТАНСА:

$$B_e = \omega C - \frac{1}{\omega L_p} \approx 0 \rightarrow \text{зато} \rightarrow \text{АЧХ ВОЗНАНИЦИ}$$

$$Y_e = G_e$$

$$Q = \frac{1}{\omega R C} = \frac{\omega L}{R} = Q_L$$

* КОМПЛЕКСНИ РАЧУН

I =

$$130. R_1 = 30 \Omega$$

$$X_1 = 40 \Omega$$

$$R_2 = 60 \Omega$$

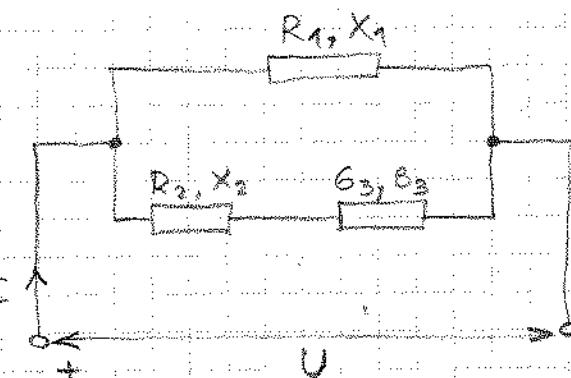
$$X_2 = 80 \Omega$$

$$G_3 = 16 mS$$

$$B_3 = 12 mS$$

$$U = 250 V$$

$$I = ?$$



$$\underline{Z}_3 = \frac{1}{G_3} = \frac{1}{16 + 12j} \cdot 10^{-3}$$

$$\underline{Z}_3 = \frac{16 - 12j}{256 + 144} k\Omega = \frac{16 - 12j}{400} k\Omega$$

$$\underline{Z}_3 = (40 - j30) \Omega$$

$$\underline{Z} = \frac{\underline{Z}_1 \underline{Z}_{23}}{\underline{Z}_1 + \underline{Z}_{23}} = \frac{\underline{Z}_1 (\underline{Z}_2 + \underline{Z}_3)}{\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_3} = \frac{\underline{Z}_1 (60 + 80j + 40 - j30)}{\underline{Z}_1 + 60 + j80 + 40 - j30}$$

$$\underline{Z} = \frac{\underline{Z}_1 \cdot (100 + j50)}{\underline{Z}_1 + 100 + j50} = \frac{(30 + j20)(100 + j50)}{30 + j200 + 100 + j50} = \frac{1000 + j5500}{130 + j30}$$

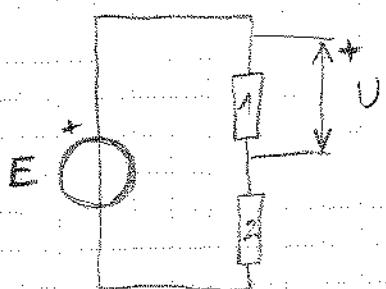
$$\underline{Z} = (25 + j25) \Omega \quad I = \frac{U}{\underline{Z}} = \frac{U}{181} = \frac{250}{\sqrt{2 \cdot 25^2}} = \frac{250}{25\sqrt{2}} = 5.52 A$$

(I = 5.52 A)

131. $\frac{U}{E} > 1$ (???)

a) $E = \text{const}$

d) $e(t)$



a) Условия для односторонних струй:

$$\frac{U}{E} = \frac{R_2}{R_1 + R_2} = \frac{R_2}{R_1 + R_2} = 1 - \frac{R_1}{R_1 + R_2} \leq 1$$

d) Условия для простоизодличных струй:

$$\frac{U}{E} = \frac{|Z_1|}{|Z_1 + Z_2|} \rightarrow \begin{aligned} \text{может быть } & \text{одинаковы} \\ \text{если } & \text{имеют одинаковые} \\ \text{карактеристики} & \end{aligned}$$

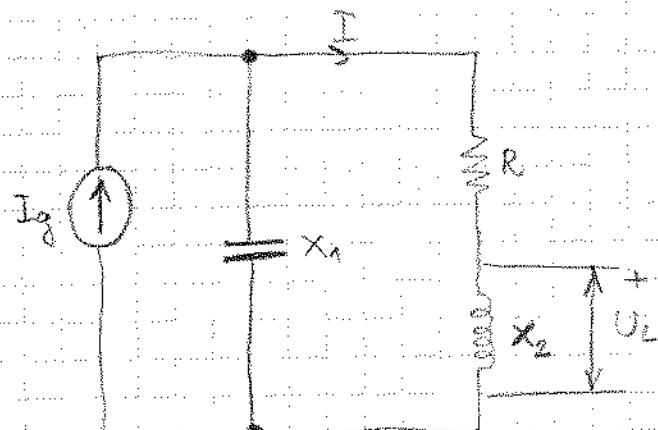
132. $I_g = 2 \mu\text{A}$

$$R = 200 \Omega$$

$$X_2 = 100 \Omega$$

$$X_1 = ?$$

$$\phi = \theta_2 - \theta_1 = 0$$



$$\frac{U_L}{U} = \frac{U_L}{U} e^{j\omega t - \phi} = \frac{U_L}{U} = \frac{ZL}{(Z_2 + ZL)Z} = \frac{X_2}{R + X_2} = \frac{100}{300} = \frac{1}{3}$$

$$U = Z I_g = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} I_g = \frac{jX_1 \cdot (R + jX_2)}{jX_1 + R + jX_2} I_g = j \cdot \frac{1}{3} I_g$$

$$U = 3 \cdot U_L$$

$$133. E = 10V$$

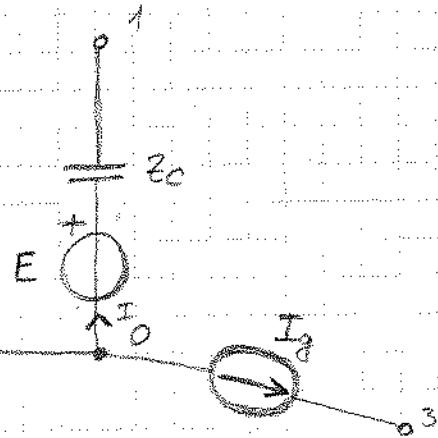
$$I_g = 2\sqrt{3}A$$

$$I_2 = 2A$$

$$Z_C = 5\Omega$$

$$\Phi_g = \theta - \Psi_g = -\frac{2\pi}{3}$$

$$\Phi_2 = \theta - \Psi_2 = -\frac{\pi}{3}$$



$$U_{10} = ?$$

• УСЕЛЯМО ПОЧЕТНУ ФОРМУ ЕЛЕКТРОМОТОРНЕ СИНЕ $\theta = 0$

$$E = E e^{j\theta} = 10 e^{j0} = 10V$$

$$\Rightarrow \Psi_g = \frac{2\pi}{3} \quad \wedge \quad \Psi_2 = \frac{\pi}{2}$$

I КИРХОФОВ ЗАКОН:

$$-I_2 + I + I_g = 0 \Rightarrow I = I_2 - I_g$$

$$I_g = I_g e^{j\Psi_g} = 2\sqrt{3} \cdot e^{j\frac{2\pi}{3}} = 2\sqrt{3} \left(-\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2} \right) A = (-\sqrt{3} + j \cdot 3) A$$

$$I_2 = I_2 e^{j\Psi_2} = 2 \cdot e^{j\frac{\pi}{2}} = j2 A$$

$$I = j2 - (-\sqrt{3} + j \cdot 3) = (\sqrt{3} - j) A$$

$$U_C = Z_C I = Z_C e^{j\theta} \cdot (\sqrt{3} - j) = 5 \cdot j \cdot (\sqrt{3} - j) V$$

$$U_C = (5\sqrt{3} j - 5) V = (-5 + j 5\sqrt{3}) V$$

$$U_{10} = E - U_C = 10 - (-5 + j 5\sqrt{3}) = (15 - j 5\sqrt{3}) V$$

$$U_{10} = |U_{10}| = \sqrt{225 + 25 \cdot 3} = \sqrt{10 \cdot 25} = 5 \cdot 2 \cdot \sqrt{3} V$$

$$U_{10} = 10\sqrt{3} V$$

134. f:

$$L = 30 \text{ mH}$$

$$C = 0.5 \mu\text{F}$$

$$X_1 = -900 \Omega$$

$$X_2 = 400 \Omega$$

$$R_2 = 400 \Omega$$

$$I_1 = 50\sqrt{2} \text{ mA}$$

$$U_{AB} = 10 \text{ V}$$

$$\Delta U_{14} = ?$$

$$f \rightarrow L_1 = 2f$$

1) на учетаности f:

$$I_g = Y_{23} U_{23}$$

$$U_{23} = |X_1| I_1 = 900 \cdot 50\sqrt{2} \cdot 10^{-3} = 20\sqrt{2} \text{ V}$$

$$G_{23} = \frac{R_2}{R_2^2 + X_2^2} = \frac{400}{2 \cdot 400^2} = \frac{1}{800} \text{ S}$$

$$B_{23} = -\frac{1}{X_1} - \frac{X_3}{R_2^2 + X_2^2} = \frac{1}{900} - \frac{1}{800} = \frac{2-1}{800} \text{ S} = \frac{1}{800} \text{ S}$$

$$Y_{23} = \sqrt{G_{23}^2 + B_{23}^2} = \sqrt{2 \cdot \frac{1}{800^2}} = \frac{\sqrt{2}}{800} \text{ S}$$

$$(I_g = Y_{23} \cdot U_{23} = \frac{\sqrt{2}}{800} \cdot 20\sqrt{2} = \frac{2}{800} = \frac{1}{400} \text{ A} = 50 \text{ mA})$$

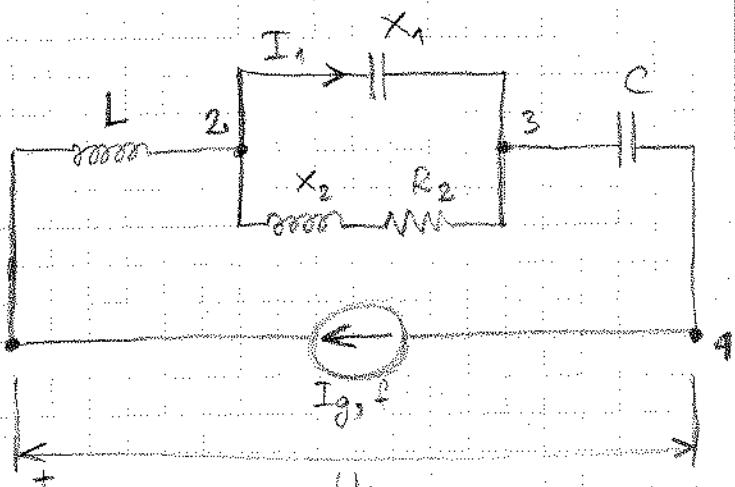
$$U_{14} = Z_{14} I_g$$

$$U_{43} = Z_C \cdot I_g \Rightarrow Z_C = \frac{U_{43}}{I_g} = \frac{10}{50 \cdot 10^{-3}} = 0.2 \text{ k}\Omega = 200 \Omega$$

$$Z_C = \frac{1}{\omega C} \Rightarrow \omega = \frac{1}{C Z_C} = \frac{1}{\frac{1}{2} \cdot 10^{-6} \cdot 200} = 10^9 \text{ s}^{-1}$$

$$Z_C = -X_C = 200 \Omega \Rightarrow X_C = -200 \Omega$$

$$X_L = \omega L = 10^9 \cdot 30 \cdot 10^{-3} \Omega = 300 \Omega$$



$$R_{14} = \frac{G_{23}}{G_{23}^2 + B_{23}^2} = \frac{\frac{1}{800}}{\left(\frac{1}{800}\right)^2 + \left(\frac{1}{300}\right)^2} = \frac{\frac{1}{800}}{\frac{1}{640000} + \frac{1}{90000}} = \frac{\frac{1}{800}}{\frac{1}{536000}} = 400 \Omega$$

$$X_{14} = X_L - \frac{B_{23}}{G_{23}^2 + B_{23}^2} + X_C = 300 - \frac{\frac{1}{300}}{\frac{1}{640000} + \frac{1}{90000}} + (-200)$$

$$X_{14} = (300 - 600) \Omega \Rightarrow X_{14} = -300 \Omega$$

$$Z_{14} = \sqrt{R_{14}^2 + X_{14}^2} = \sqrt{400^2 + 300^2} = \sqrt{1600^2 (16+9)} = \sqrt{1600^2 \cdot 25} = 1600 \cdot 5 = 8000 \Omega$$

$$Z_{14} = 1600 \Omega = 500 \Omega$$

$$U_{14} = Z_{14} I_a = 500 \cdot 50 \cdot 10^{-3} = 25 V$$

$$\boxed{U_{14} = 25 V}$$

2) на участности 2f: $I_a = 2 A$

$$G_{23}^{(1)} = \frac{R_2}{R_2^2 + (2X_2)^2} = \frac{900}{1600 + 4 \cdot 1600} = \frac{900}{5 \cdot 1600} = \frac{1}{2} \cdot 10^{-3} \Omega$$

$$G_{23}^{(1)} = 0.5 \mu S$$

$$B_{23}^{(1)} = -\frac{1}{2} \frac{2X_2}{R_2^2 + (2X_2)^2} = -\frac{X_2}{1600} = \frac{800}{5 \cdot 1600} = \frac{1}{200} = \frac{1}{5} \cdot \frac{1}{200} \Omega$$

$$B_{23}^{(1)} = \frac{1}{5} \frac{1}{200} = \frac{1}{250} S = \frac{9}{1000} S = 9 \mu S \Rightarrow B_{23}^{(1)} = 9 \mu S$$

$$R_{14}^{(1)} = \frac{G_{23}^{(1)}}{G_{23}^{(1)2} + B_{23}^{(1)2}} = \frac{0.5 \cdot 10^{-3}}{\frac{1}{4} \cdot 10^{-6} + 16 \cdot 10^{-6}} = \frac{1}{55} \cdot 10^{-3} = \frac{1}{65} k\Omega$$

$$R_{23}^{(1)} = R_{14}^{(1)} = \frac{3}{65} k\Omega$$

$$X_{23}^{(1)} = \frac{-B_{23}^{(1)}}{(B_{23}^{(1)})^2 + (G_{23}^{(1)})^2} = \frac{-9 \cdot 10^{-3}}{16 \cdot 10^{-6} + \frac{1}{4} \cdot 10^{-6}} = \frac{-9}{65} = -\frac{9}{65} \cdot 10^3 \Omega$$

$$X_{23}^{(1)} = -\frac{16}{65} k\Omega$$

$$X_{14}^{(1)} = 2X_L + X_{23}^{(1)} + \frac{X_C}{2} = 600 - \frac{16}{65} \cdot 10^3 - \frac{200}{2} = 500 - \frac{16}{65} \cdot 10^3 \Omega$$

$$X_{14}^{(1)} = -\frac{16.5}{65} k\Omega$$

$$Z_{14}^{(1)} = \sqrt{R_{14}^{(1)2} + X_{14}^{(1)2}} = \sqrt{\frac{256,25}{65}} \approx 256 \Omega$$

$$U_{14}^{(1)} = Z_{14}^{(1)} I_a = 256 \cdot 50 \cdot 10^{-3} \approx 12.8 V \Rightarrow \Delta U_{14} = U_{14}^{(1)} - U_{14} = 12.8 - 25 = -12.2 V$$

135. $U = 12V$

$$\omega = 10^3 \text{ s}^{-1}$$

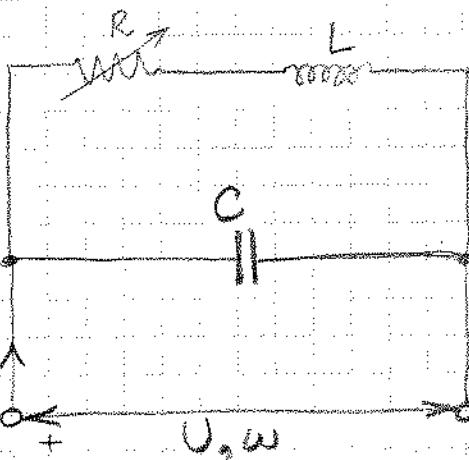
$$L = 0.9 \text{ H}$$

R ПРОМЕНЯЕТСЯ

a) $C = ?$

I не зависит от R

d) $I = ?$



$$a) Z = Y_C + Y_{RL} = j\omega C + \frac{1}{Z_{RL}} = j\omega C + \frac{1}{R + j\omega L}$$

$$Y = \frac{j\omega CR - \omega^2 LC + 1}{R + j\omega L} \cdot \frac{R - j\omega L}{R - j\omega L}$$

$$Y = j\omega CR^2 - \omega^2 LCR + R + \omega^2 LC + j\omega^3 L^2 C - j\omega L$$

$$Y = \frac{R + j(\omega CR^2 + \omega^2 L^2 C - \omega L)}{R^2 + (\omega L)^2}$$

$$|\Sigma| = Y = \sqrt{\frac{R^2}{(R^2 + (\omega L)^2)^2} + \frac{(R^2 + (\omega L)^2)^2}{(R^2 + (\omega L)^2)^2}}$$

136. $I_c = 2A$

$$I_{R1} = \sqrt{3} I_c$$

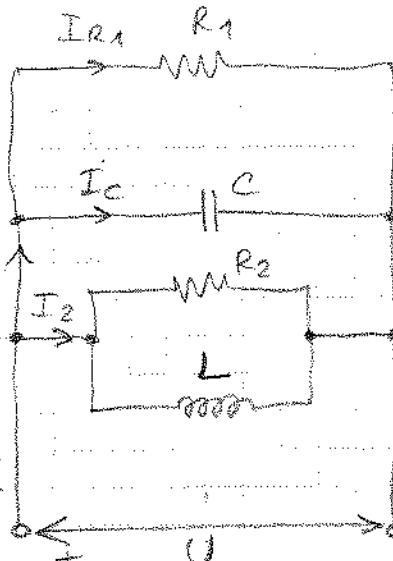
$$I_2 = 2\sqrt{3} A$$

$$\angle = \theta - \phi_2 = \frac{\pi}{3}$$

a) $I = ?$

b) $\phi = \theta - \psi = ?$

$$\phi_2 = \angle + \frac{\pi}{3}$$



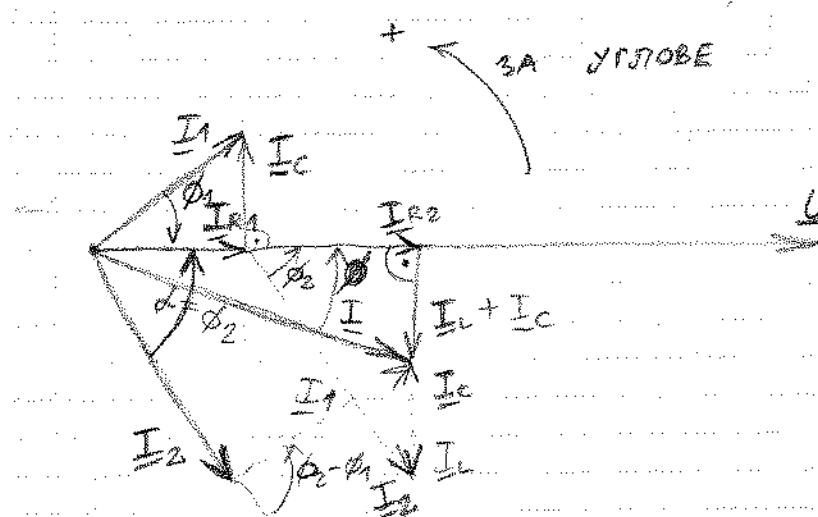
I

U

I_1

I_2

I



$$\phi_1 = -\arctan \frac{I_c}{I_{R1}} = -\arctan \frac{2}{2\sqrt{3}} = -\frac{\pi}{6}$$

$$\angle = \phi_2 = \frac{\pi}{3} = \theta - \phi_2$$

$$I_1 = \sqrt{I_{R1}^2 + I_c^2} = \sqrt{4.3 + 4} = \sqrt{9.9} = 3A$$

$$I^2 = I_1^2 + I_2^2 - 2 \cdot I_1 \cdot I_2 \cos(\theta - (\phi_2 - \phi_1))$$

$$I^2 = I_1^2 + I_2^2 + 2 \cdot I_1 \cdot I_2 \cos(\phi_2 - \phi_1)$$

$$I^2 = I_1^2 + I_2^2 + 2 \cdot I_1 \cdot I_2 \cos \frac{\pi}{3} - (-\frac{\pi}{6}) = \frac{5}{2}$$

$$I = \sqrt{I_1^2 + I_2^2} =$$

d) $\phi = \arctan \frac{I_L - I_c}{I_{R1} + I_{R2}}$

$$I_L = I_2 \sin \phi_2 = 2\sqrt{3} \sin \frac{\pi}{3} = 2\sqrt{3} \frac{\sqrt{3}}{2} = 3A$$

$$I_{R2} = I_2 \cos \phi_2 = 2\sqrt{3} \cos \frac{\pi}{3} = 2\sqrt{3} \frac{1}{2} = \sqrt{3} A$$

$$\phi = \arctan \frac{3 - 2}{\sqrt{3} + \sqrt{3}} = \arctan \frac{1}{3\sqrt{3}}$$

2) в комплексном домене:

a) убедимся за почету фазы напона $\phi = 0$

$$I_{R1} = I_{R1} e^{j\omega t} = I_{R1}$$

$$I_C = I_C \cdot e^{j\frac{\pi}{2}} =$$

$$I_2 = I_2 e^{j\omega^2} = I_2 e^{j(0-3)} = I_2 e^{-j\pi} = I_2 e^{-j\frac{\pi}{2}}$$

$$I = I_{R1} + I_C + I_2 =$$

$$I = |I| =$$

$$\phi = -\arg(I) = -\arg($$

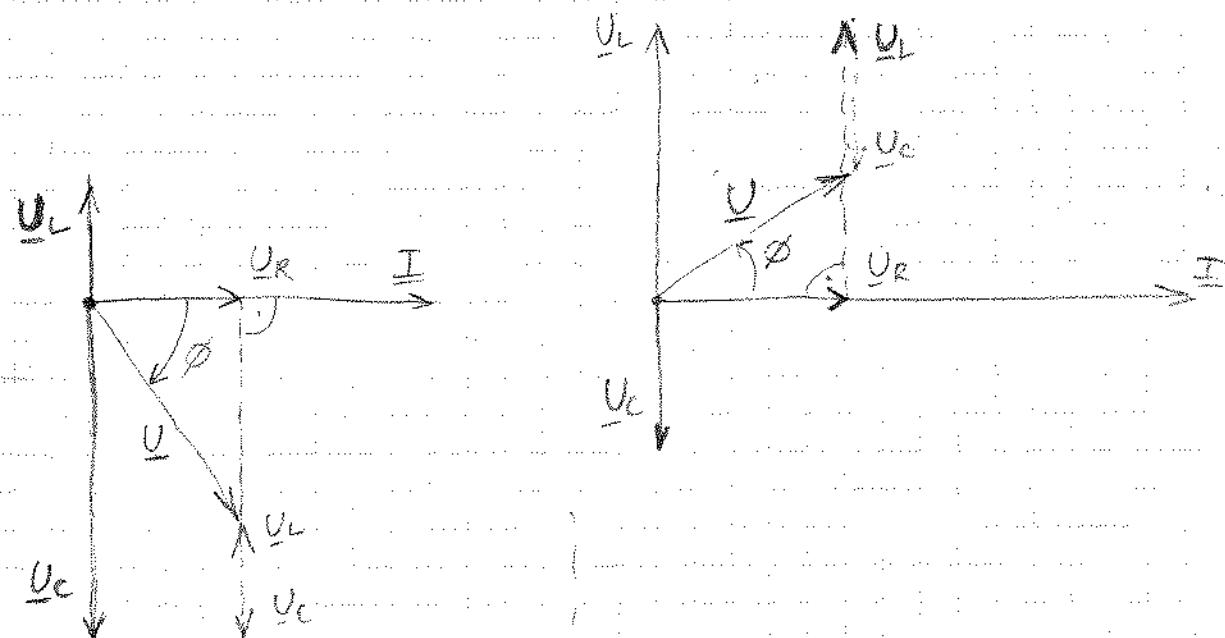
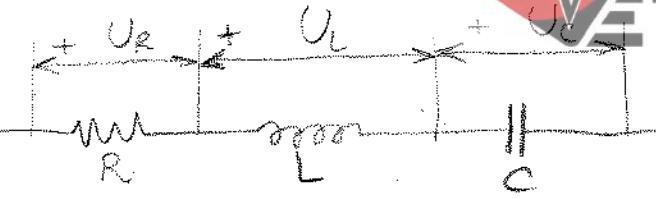
137. $U_R = 40 \text{ V}$

$U_c = 90 \text{ V}$

$U = 50 \text{ V}$

a) $U_L = ?$

d) $\phi = \theta - \psi = ?$



a) $U = U_R + U_L + U_C$ $U = U_R + U_L + U_C$

$$\Rightarrow U^2 = U_R^2 + (U_C - U_L)^2 \quad U^2 = U_R^2 + (U_L - U_C)^2$$

$$(U_C - U_L)^2 = U^2 - U_R^2 \quad (U_L - U_C)^2 = U^2 - U_R^2$$

$$U_C - U_L = \sqrt{U^2 - U_R^2} \quad U_L - U_C = \sqrt{U^2 - U_R^2}$$

$$U_L^{(1)} = U_C - \sqrt{U^2 - U_R^2} \quad U_L^{(2)} = U_C + \sqrt{U^2 - U_R^2}$$

$$U_C^{(1)} = 90 - \sqrt{50^2 - 40^2} \quad U_L^{(2)} = 90 + \sqrt{50^2 - 40^2}$$

$$U_L^{(1)} = 90 - \sqrt{10^2 (5^2 - 4^2)} \quad U_L^{(2)} = 90 + \sqrt{10^2 \cdot 9}$$

$$U_C^{(1)} = 90 - \sqrt{10^2 \cdot 9} \quad U_C^{(2)} = 90 + 10 \cdot 3$$

$$U_C^{(1)} = 90 - 10 \cdot 3 \quad U_L^{(2)} = (90 + 30) \text{ V}$$

$$(U_L^{(1)} = (90 - 30) \text{ V} = 60 \text{ V}) \quad (U_C^{(2)} = 120 \text{ V})$$

3) ФАЗНАЯ РАЗЛИКА $U \neq I$

$$\phi = \arctg \frac{U_L - U_C}{U_R}$$

$$\phi^{(1)} = \arctg \frac{-1(U_L - U_C)}{U_R} = \arctg \frac{-3\phi}{4\phi} = \arctg \frac{-3}{4} \approx -0.69 \text{ rad} \approx -36.9^\circ$$

$$\phi^{(2)} = \arctg \frac{U_L - U_C}{U_R} = \arctg \frac{12\phi - 3\phi}{4\phi} = \arctg \frac{3}{4} \approx 0.69 \text{ rad} \approx 36.9^\circ$$

2) в комплексном домене:

$$U = U_R + U_L + U_C$$

$$I = I e^{j\psi}$$

$$\Rightarrow U = U e^{j\psi} = U e^{j(\psi + \phi)}$$

$$U_R = U_R e^{j\psi}$$

$$U_L = U_L e^{j(\psi + \frac{\pi}{2})}$$

$$U_C = U_C e^{j(\psi - \frac{\pi}{2})}$$

$$\Rightarrow$$

$$\Rightarrow U e^{j(\psi + \phi)} = U_R e^{j\psi} + U_L e^{j(\psi + \frac{\pi}{2})} + U_C e^{j(\psi - \frac{\pi}{2})}$$

$$U e^{j\psi} \cdot e^{j\phi} = U_R e^{j\psi} + U_L e^{j\psi} \cdot j + U_C e^{j\psi} \cdot (-j) \quad /: e^{j\psi}$$

$$U e^{j\phi} = U_R + j U_L - j U_C = U_R + j (U_L - U_C)$$

$$U^2 = U_R^2 + (U_L - U_C)^2 \quad \arg(U e^{j\phi}) = \arctg \frac{U_L - U_C}{U_R}$$

$$\Rightarrow \boxed{U_L = U_C \pm \sqrt{U^2 - U_R^2}} \quad \Rightarrow \quad \phi = \arctg \frac{U_L - U_C}{U_R}$$

$$\Rightarrow \boxed{\operatorname{tg} \phi = \frac{U_L - U_C}{U_R}}$$