Стефан Ћировић

ОСНОВИ РАЧУНАРСКЕ ТЕХНИКЕ

• РЕШЕНИ КОЛОКВИЈУМИ ’08-’11 •

Београд
2013.

I КОЛОКВИЈУМ 13.09.2008

Датум

1) -a)

$$f(x_1,x_2,x_3,x_4) = \bar{x}_1 \cdot \bar{x}_3 \cdot x_4 + \bar{x}_1 x_2 \bar{x}_3 + x_2 x_3 x_4 + x_1 \bar{x}_2 + x_1 \bar{x}_4$$

$$g(x_1,x_2,x_3,x_4) = (\bar{x}_1 + \bar{x}_2 + \bar{x}_4) \cdot (x_2 + x_3 + x_4) \cdot (x_1 + \bar{x}_3 + x_4)$$

$f(1) = \{0x01, 010x, x111, 10xx, 1xx0\}$

$g(0) = \{11x1, x000, 0x10\}$

$f(1) = \{0001, 0101, 0100, 0101, 0111, 1111, 1000, 1001, 1010, 1011, 1000, 1010, 1100, 1110\}$

$g(0) = \{1101, 1111, 0000, 1000, 0010, 0110\}$

$f(1) = \{1, 5, 4, 7, 15, 8, 9, 10, 11, 12, 14\}$

$g(0) = \{13, 15, 0, 8, 2, 6\}$

$f(1) = g(0)$ за $\{8, 3, 15\}$

i	x_1	x_2	x_3	x_4	f	g
0	0	0	0	0	0	0
1	0	0	0	1	1	1
2	0	0	1	0	0	0
3	0	0	1	1	0	1
4	0	1	0	0	1	1
5	0	1	0	1	1	1
6	0	1	1	0	0	0
7	0	1	1	1	1	1
8	1	0	0	0	1	0
9	1	0	0	1	1	1
10	1	0	1	0	1	1
11	1	0	1	1	1	1
12	1	1	0	0	1	1
13	1	1	0	1	0	0
14	1	1	1	0	1	1

1 0

НИС

Датум .....

б) $f(x_1, x_2, x_3, x_4) = (x_1 + \overline{x_2} \cdot \overline{x_3}) \cdot (\overline{x_2 + x_3 \cdot x_4}) + \overline{x_3} + x_2$

$f = (x_1 + \overline{x_2} \cdot \overline{x_3}) \cdot (\overline{x_2} \cdot (\overline{x_3} + \overline{x_4})) + \overline{x_3} + x_2$

$f = (x_1 + \overline{x_2} \cdot \overline{x_3}) \cdot (\overline{x_2} \cdot \overline{x_3} + \overline{x_2} \overline{x_4}) + \overline{x_3} + x_2$

$f = x_1 \overline{x_2} \overline{x_3} + x_1 \overline{x_2} \overline{x_4} + \overline{x_2} \overline{x_3} + \overline{x_2} \overline{x_3} \overline{x_4} + \overline{x_3} + x_2$

$f = \overline{x_3} + \overline{x_2} \overline{x_3} + \overline{x_2} \overline{x_3} \overline{x_4} + x_1 \overline{x_2} \overline{x_4} + x_2$

$f = x_1 \overline{x_2} \overline{x_4} + \overline{x_3} + x_2$ ДНФ

$f(1) = \{10x0, xx0x, x1xx\}$

$f(1) = \{1000, 1010, 0000, 0001, 0100, 0101, 1000, 1001, 1100, 1101, 0100, 0101, 0110, 0111, 1100, 1101, 1110, 1111\}$

$f(1) = \{8, 10, 0, 1, 4, 5, 9, 12, 13, 6, 7, 14, 15\}$

$f(1) = \{0, 1, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15\}$

$f(0) = \{2, 3, 11\}$

$f(0) = \{0010, 0011, 1011\}$

СКНФ: $f(x_1, x_2, x_3, x_4) = (x_1 + x_2 + \overline{x_3} + x_4) \cdot (x_1 + x_2 + \overline{x_3} + \overline{x_4}) \cdot (\overline{x_1} + x_2 + \overline{x_3} + \overline{x_4})$

45 min

НИС

Датум

2. a) $\boxed{\text{MIN KNF}}$

$$f(x_1,x_2,x_3,x_4)=(x_1+x_2+\overline{x_3})\left(\overline{\overline{x_1}+\overline{x_4}\cdot\left(\overline{x_2+x_3\cdot\overline{x_4}}\right)}\right)$$

$$f=(x_1+x_2+\overline{x_3})\left(\overline{\overline{x_1}+\overline{x_4}\cdot\left(\overline{x_2}\cdot(\overline{x_3}+x_4)\right)}\right)$$

$$f=(x_1+x_2+\overline{x_3})\left(\overline{\overline{x_1}+\overline{x_4}\left(\overline{x_2}\,\overline{x_3}+\overline{x_2}x_4\right)}\right)$$

$$f=(x_1+x_2+\overline{x_3})\left(\overline{\overline{x_1}+\overline{x_2}\,\overline{x_3}\,\overline{x_4}}\right)$$

$$f=(x_1+x_2+\overline{x_3})\left(x_1\cdot\left(\overline{\overline{x_2}\,\overline{x_3}\,\overline{x_4}}\right)\right)$$

$$f=(x_1+x_2+\overline{x_3})\cdot\left(x_1\cdot(x_2+x_3+x_4)\right)$$

$$f=(x_1+x_2+\overline{x_3})(x_1x_2+x_1x_3+x_1x_4)$$

$$f=x_1x_2+x_1x_3+\boxed{x_1x_4}+\cancel{x_1x_2}+\cancel{x_1x_2x_3}+\cancel{x_1x_2x_4}+\cancel{x_1x_2\overline{x_3}}+\cancel{x_1\overline{x_3}x_4}$$

$f=x_1x_2+x_1x_3+x_1x_4$ ДНФ

$f(1)=\{11xx,\ 1x1x,\ 1xx1\}$

$f(1)=\{1100,\ 1101,\ 1110,\ 1111,\ 1010,\ 1011,\ \cancel{1110},\ \cancel{1111},\ 1001,\ \cancel{1011},\ \cancel{1101},\ \cancel{1111}\}$

$f(1)=\{12,\ 13,\ 14,\ 15,\ 10,\ 11,\ 9\}$

x_3x_4 \ x_1x_2	00	01	11	10
00	0_0	0_4	1_{12}	0_8
01	0_1	0_5	1_{13}	1_9
11	0_3	0_7	1_{15}	1_{11}
10	0_2	0_6	1_{14}	1_{10}

KNF MIN

$$\boxed{f(x_1,x_2,x_3,x_4)=x_1\cdot(x_2+x_3+x_4)}$$

НИС

Датум .....

MIN DNF

(2. б)) $f(x_1, x_2, x_3) = (\overline{x_2} + x_3) \cdot (\overline{x_1} + \overline{x_2}) \cdot (x_1 + x_2 + \overline{x_3})$

$f = (\overline{x_2} + x_3)(\overline{x_1}x_2 + \overline{x_1}\overline{x_3} + x_1\overline{x_2} + \overline{x_2}\overline{x_3})$

$f = \overline{x_1}\overline{x_2}\overline{x_3} + x_1\overline{x_2} + \overline{x_2}\overline{x_3} + \overline{x_1}x_2x_3 + x_1\overline{x_2}x_3$

$f = x_1\overline{x_2} + \overline{x_2}\overline{x_3} + \overline{x_1}x_2x_3$ DNF

$f(1) = \{10x, x00, 011\} = \{100, 101, 000, 100, 011\}$

$f(1) = \{4, 5, 0, 3\}$

x_3 \ x_1x_2	00	01	11	10
0	1 0	2	6	1 4
1	1	1 3	7	1 5

MIN DNF

$f(x_1, x_2, x_3) = \overline{x_2}\overline{x_3} + \overline{x_1}x_2x_3 + x_1\overline{x_2}$

2. в)) $f(x_1, x_2, x_3, x_4)$ MIN DNF

$f(0) = \{1, 9, 13, 19, 15\}$

x_3x_4 \ x_1x_2	00	01	11	10
00	1 0	0 4	1 12	1 8
01	0 1	1 5	0 13	1 9
11	1 3	1 7	0 15	1 11
10	1 2	1 6	0 14	1 10

$f(x_1, x_2, x_3, x_4) = x_1\overline{x_2} + \overline{x_1} \cdot x_3 + \overline{x_1}x_2x_4 +$

MIN DNF $x_1\overline{x_3}\overline{x_4} + \overline{x_2}\overline{x_4}$

Датум

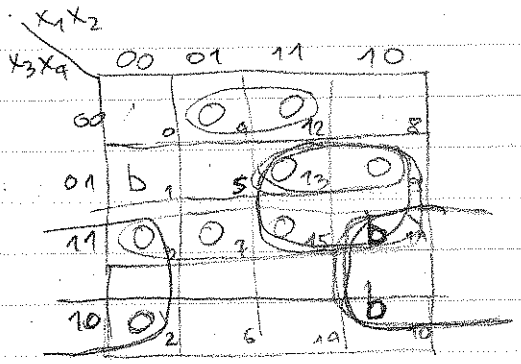
MIN KNF

2. г) $f(x_1, x_2, x_3, x_4)$

$f(1) = \{0, 5, 6, 8, 14\}$

$f(b) = \{1, 10, 11\}$

x_3x_4 \ x_1x_2	00	01	11	10
00	0	0 4	0 12	8
01	b 1	5	0 13	0 9
11	0 3	0 7	0 15	b 11
10	0 2	6	14	b 10



$$f(x_1, x_2, x_3, x_4) = (\overline{x_3} + \overline{x_4})(\overline{x_2} + x_3 + x_4)(x_2 + \overline{x_3})(\overline{x_1} + \overline{x_4})$$

I КОЛОКВИЈУМ 01.06.2008.

НИС НАДОКНАДА Датум

1. а) $f(x_1, x_2, x_3, x_4)$

$f(1) = \{1, 3, 9, 8, 9, 10, 12, 13, 15\}$

$f(b) = \{2, 6, 7\}$

$b = ?$ → SKNF = MIN KNF

x_3x_4 \ x_1x_2	00	01	11	10
00	0 (0)	1 (4)	1 (12)	1 (8)
01	1 (1)	0 (5)	1 (13)	1 (9)
11	1 (3)	b (7)	1 (15)	0 (11)
10	b (2)	b (6)	0 (14)	1 (10)

x_3x_4 \ x_1x_2	00	01	11	10
00	0			
01		0		
11		b		0
10	b	b	0	

$f(0) = \{0, 5, 11, 14\}$

$f(1) = f(b) = \{2, 6, 7\}$ KADA SKNF = MIN KNF

б) $f(x_1, x_2, x_3, x_4) \rightarrow 0$

$f = (\overline{x_1} \cdot \overline{x_2} + x_3 \overline{x_4} \cdot \overline{(x_1 + \overline{x_2} \cdot x_3 \cdot (x_2 + x_1 \cdot \overline{x_4}))})$

$f = (\overline{x_1} \cdot \overline{x_2} + x_3 \cdot \overline{x_4} \cdot \overline{(x_1 + \overline{x_2} \cdot x_3 \cdot \overline{x_2} \cdot \overline{(x_1 + x_4)})})$

$f = (\overline{\overline{x_1}\overline{x_2} + x_3\overline{x_4}} \cdot \overline{x_1} \cdot (x_2 + \overline{x_3} + \overline{x_2} + x_1\overline{x_4}))$

$f = (\overline{\overline{x_1}\overline{x_2} + \overline{x_1}x_2x_3\overline{x_4}}) = (x_1 + x_2)(x_1 + \overline{x_2} + \overline{x_3} + x_4)$

$f(0) = \{00xx, 0110\} = \{0000, 0001, 0010, 0011, 0101\}$

$f(0) = \{0, 1, 2, 3, 6\}$

Датум

MIN KNF

2. a) $f(x_1,x_2,x_3,x_4)=(x_1+\bar{x}_2\cdot\bar{x}_3+x_4(\overline{x_1+\bar{x}_2\bar{x}_3\bar{x}_4})\cdot(x_3+\bar{x}_1\bar{x}_2))$

$f=(x_1+\bar{x}_2\bar{x}_3+x_4\bar{x}_1\cdot(x_2+x_3+x_4)\cdot(x_3+\bar{x}_1\bar{x}_2))$

$f=x_1+\bar{x}_2\bar{x}_3+(\bar{x}_1x_2x_4+\bar{x}_1x_3x_4+\bar{x}_1x_4)(x_3+\bar{x}_1\bar{x}_2)$

$f=x_1+\bar{x}_2\bar{x}_3+\bar{x}_1x_2x_3x_4+\bar{x}_1x_3x_4+\bar{x}_1x_3x_4+\bar{x}_1x_2\bar{x}_3x_4+\bar{x}_1\bar{x}_2x_4$

$f=x_1+\bar{x}_2\bar{x}_3+\bar{x}_1x_3x_4+\bar{x}_1\bar{x}_2x_4$

x_3x_4 \ x_1x_2	00	01	11	10
00	1	0	1	1
01	1	0	1	1
11	1	1	1	1
10	0	0	1	1

$f(1)=\{1xxx, x00x, 0x11, 00x1\}$

$f(1)=\{1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111, 0000, 0001, 1000, 1001, 0011, 0111, 0001, 0011\}$

$f(1)=\{8,9,10,11,12,13,14,15,0,1,3,7\}$

$f(0)=\{2,9,5,6\}$

$f(x_1,x_2,x_3,x_4)=(x_1+\bar{x}_2+x_3)(x_1+\bar{x}_3+x_4)$

НИС Datum

MIN DNF

(2. d) $f(x_1, x_2, x_3) = \bar{x}_1 x_2 \bar{x}_3 + x_2 x_3 + x_1 \bar{x}_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 x_3$

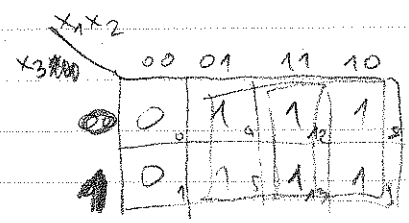
$f(1) = \{010, x11, 1x0, 100, 1x1\}$

$f(1) = \{010, 011, 111, 100, 110, 100, 101, 111\}$

$f(1) = \{2, 3, 7, 9, 6, 5\}$

$f(0) = \{0, 1\}$

$f(x_1, x_2, x_3) = x_2 + x_1$



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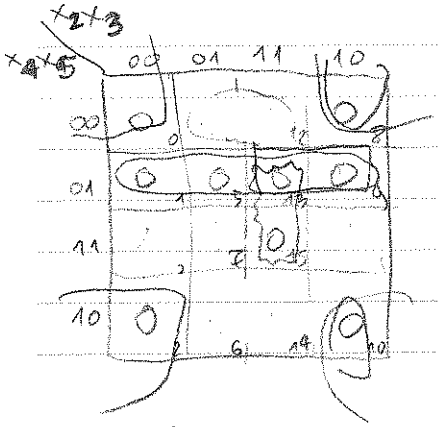
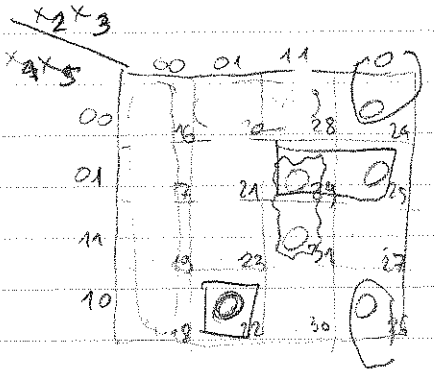
b) MIN KNF $f(x_1, x_2, x_3, x_4, x_5)$

$f(1) = \{3, 4, 6, 7, 11, 12, 14, 16, 17, 18, 19, 20, 21, 23, 27, 28, 30\}$

$f(0) = \{0, 1, 2, 5, 8, 9, 10, 13, 15, 22, 24, 25, 26, 29\}$



$x_1 = 0$



$x_1 = 1$

НИС

Датум .....

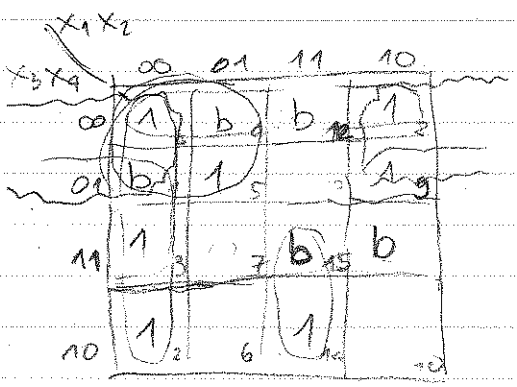
$f(x_1, x_2, x_3, x_4, x_5) = (\overline{x_2} + \overline{x_3} + \overline{x_5})(\overline{x_2} + x_4 + \overline{x_5}) \cdot (\overline{x_2} + x_3 + x_5)(x_1 + x_4 + \overline{x_5})(x_1 + x_3 + x_5)(\overline{x_1} + x_2 + \overline{x_3} + \overline{x_4} + x_5)$

2. i) $f(x_1, x_2, x_3, x_4)$ MIN DNF ?

$f(0) = \{6, 7, 10, 13\}$

$f(b) = \{1, 9, 11, 12, 15\}$

$f(1) = \{0, 2, 3, 5, 8, 9, 14\}$



$f(x_1, x_2, x_3, x_4) = \overline{x_1}\overline{x_2} + \overline{x_2}\overline{x_3} + \overline{x_1}\overline{x_3} + \ldots + x_1 x_2 x_3$

I КОЛОКВИЈУМ 28.03.2009.

ПРВИ Датум

1. а) $f(x_1, x_2, x_3, x_4) = \overline{x_1}\, x_2 \cdot x_3 + x_2 \overline{x_3}\, x_4 + x_1 x_2 \overline{x_3}$

$g(x_1, x_2, x_3, x_4) = (x_1 + \overline{x_2} + \overline{x_3})(x_2 + \overline{x_4})(\overline{x_1} + x_3 + x_4)(x_2 + x_4)$

$f(1) = \{011x, x101, 110x\}$

$f(1) = \{0110, 0111, 0101, 1101, 1100, 1101\}$

$f(1) = \{6, 7, 5, 13, 12\}$

$g(0) = \{011x, x0x1, 1x00, x0x0\}$

$g(0) = \{0110, 0111, 0001, 0011, 1001, 1011, 1000, 1100, 0000, 0010, 1000, 1010\}$

$g(0) = \{6, 7, 1, 3, 9, 11, 8, 12, 0, 2, 10\}$

i	x_1	x_2	x_3	x_4	f	g
0					0	0
1					0	0
2					0	0
3					0	0
4					0	1 ←
5					1	1
6					1	0 ←
7					1	0 ←
8					0	0
9					0	0
10					0	0
11					0	0
12					1	0 ←
13					1	1
14					0	1 ←
15					0	1 ←

$\{4, 6, 7, 12, 14, 15\}$

НИС Датум

д) (SKNF ???)

$f(x_1, x_2, x_3, x_4) = x_1\bar{x}_4 + \overline{((x_1 + x_2 + \bar{x}_3 \cdot (x_1 + \bar{x}_4)) + x_2 \cdot x_3)} + \bar{x}_1 \cdot \bar{x}_2$

$f = x_1\bar{x}_4 + \overline{((\bar{x}_1 \cdot \bar{x}_2 \cdot (x_3 + \bar{x}_1 \cdot x_4)) + x_2 x_3)} + \bar{x}_1 \cdot \bar{x}_2$

$f = x_1\bar{x}_4 + (((x_1 + x_2 + \bar{x}_3 (x_1 + \bar{x}_4)) \cdot (\bar{x}_2 + \bar{x}_3)) + \bar{x}_1\bar{x}_2$

$f = x_1\bar{x}_4 + x_1\bar{x}_2 + x_1\bar{x}_3 + x_2\bar{x}_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_3 + \bar{x}_2\bar{x}_3\bar{x}_4 + \bar{x}_3\bar{x}_4 + \bar{x}_1\bar{x}_2$

$f = x_1\bar{x}_4 + x_1\bar{x}_2 + x_1\bar{x}_3 + x_2\bar{x}_3 + \bar{x}_3\bar{x}_4 + \bar{x}_1\bar{x}_2$

$f(1) = \{1xx0, 10xx, 1x0x, x10x, xx00, 00xx\}$

$f(1) = \{1000, 1010, 1100, 1110, 1000, 1001, 1010, 1011, 1000, 1001, 1100, 1101, 0100, 0101, 1100, 1101, 0000, 0100, 1000, 1100, 0000, 0001, 0010, 0011\}$

$f(1) = \{8, 10, 12, 14, 9, 11, 13, 9, 5, 0, 1, 2, 3\}$

$f(0) = \{6, 7, 15\}$

$f(0) = \{0110, 0111, 1111\}$

(SKNF)

$f(x_1x_2, x_3, x_4) = (x_1 + \bar{x}_2 + \bar{x}_3 + x_4)(x_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4) \cdot (\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + x_4)$

НИС

Датум .....

MIN KNF

2. a) $f = \left(x_1 + \left(\overline{x_2}\,\overline{x_3}\,(x_2 + \overline{x_4}) + x_1(\overline{x_3 + x_4})\right)\right) \cdot (x_1 + \overline{x_3} \cdot x_4)$

$f = \left(x_1 + \left(\overline{x_2}\,\overline{x_3} \cdot \overline{x_2} \cdot x_4 + x_1 \cdot \overline{x_3} \cdot \overline{x_4}\right)\right)(x_1 + \overline{x_3} x_4)$

$f = \left(x_1 + (x_2 + x_3 + \overline{x_4}) \cdot (\overline{x_1} + x_3 + x_4)\right)(x_1 + \overline{x_3} x_4)$

$f = (x_1 + \overline{x}_1 x_2 + x_2 x_3 + x_2 x_4 + \overline{x}_1 x_3 + x_3 + x_3 x_4 + \overline{x}_1 \overline{x}_4 + x_3 \overline{x}_4)(x_1 + \overline{x}_3 x_4)$

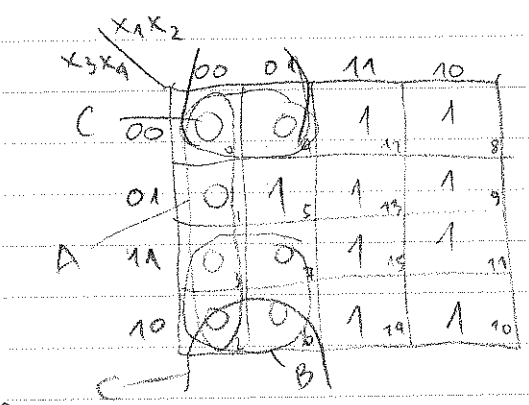
$f = x_1 + \overline{x}_1 x_2 \overline{x}_3 x_4 + \overline{x}_1 x_2 \overline{x}_3 x_4 + x_1 x_2 \overline{x}_3 + x_1 x_2 x_4 + x_2 \overline{x}_3 x_4 + x_2 x_3 + x_1 x_3 x_4 + x_1 \overline{x}_3 x_4$

$f = x_1 + \overline{x}_1 x_2 \overline{x}_3 x_4 + x_2 \overline{x}_3 x_4$

$f(1) = \{1xxx, 0101, x101\}$

$f(1) = \{1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111, 0101, 1101, 0101\}$

$f(1) = \{8, 9, 10, 11, 12, 13, 14, 15, 5\}$



$f(x_1, x_2, x_3, x_4) = A \cdot B \cdot C = (x_1 + x_2)(x_1 + \overline{x_3})(x_1 + x_4)$

$A = x_1 + x_2$

$B = x_1 + \overline{x_3}$

$C = x_1 + x_4$

НИС Датум

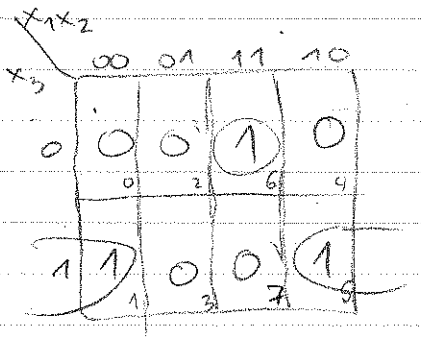
MIN DNF ???

2. a) $f(x_1,x_2,x_3) = (x_1+x_3)(\overline{x_2}+\overline{x_3})\cdot(\overline{x_1}+x_2+x_3)$

$f(0) = \{0x0, x11, 100\}$

$f(0) = \{000, 010, 011, 111, 100\}$

$f(0) = \{0, 2, 3, 7, 4\}$



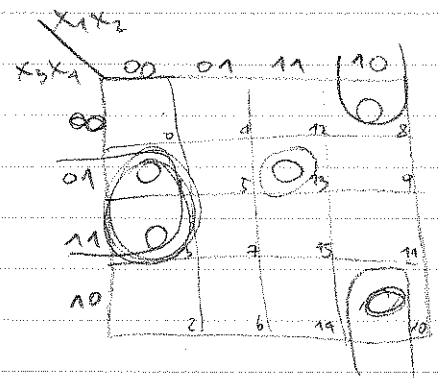
$f(x_1,x_2,x_3) = \overline{x_2}x_3 + x_1x_2\overline{x_3}$

2. b) MIN KNF???

$f(x_1,x_2,x_3,x_4)$

$f(0) = \{1, 3, 8, 10, 13\}$

$f(1) = \{0, 2, 4, 5, 6, 7, 9, 11, 12, 14, 15\}$



$f(x_1,x_2,x_3,x_4) =$

$(x_1+x_2+\overline{x_4})(\overline{x_1}+x_2+x_4)$

$(\overline{x_1}+\overline{x_2}+x_3+\overline{x_4})$

Датум

MIN DNF ???

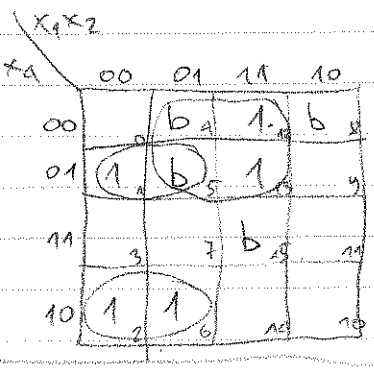
2. ā)

$f(x_1, x_2, x_3, x_4)$

$f(1) = \{1, 2, 6, 12, 13\}$

$f(b) = \{4, 5, 8, 15\}$

$f(0) = \{0, 3, 7, 9, 10, 11, 14\}$



$f(x_1, x_2, x_3, x_4) = x_2 \cdot \bar{x}_3 + \bar{x}_1 \cdot \bar{x}_3 x_4 + \bar{x}_1 \cdot x_3 \bar{x}_4$

I КОЛОКВИЈУМ 16.05.2009.

НИС НАДОКНАДА Датум

1. а) $f(x_1, x_2, x_3, x_4)$

$f(1) = \{0, 2, 5, 9, 10, 12, 15\}$

$f(b) = \{3, 6, 13\}$

SKNF = MIN KNF

x_3x_4 \ x_1x_2	00	01	11	10
00	1 (0)	(4)	1 (12)	(8)
01	(1)	1 (5)	b (13)	1 (9)
11	b (3)	(7)	1 (15)	(11)
10	1 (2)	b (6)	(14)	1 (10)

x_3x_4 \ x_1x_2	00	01	11	10
00		0		0
01	0		b	
11	b	0		0
10		b	0	

1) $f(1) = \{3, 6\}\ f(0) = \{13\} \rightarrow$ SKNF = MIN KNF

2) $f(1) = \{3, 6, 13\} \rightarrow$ SKNF = MIN KNF

б) $f(x_1, x_2, x_3, x_4) = (x_1 + x_2 + \overline{x_3})(\overline{x_1} + x_3) \cdot (x_3 + x_4)$

$g(x_1, x_2, x_3, x_4) = \overline{x_3}\,\overline{x_4} + \overline{x_1}\,\overline{x_2}x_3 + x_1 \cdot \overline{x_3}$

$f(0) = \{0010, 00x1, 1x0x, xx00\}$

$f(0) = \{0010, 0011, 1000, 1001, 1100, 1101, 0000, 0100, 1000, 1100\}$

$f(0) = \{0, 2, 3, 9, 8, 9, 12, 13\}$ $f(1) = \{1, 5, 6, 7, 10, 11, 14, 15\}$

$g(1) = \{xx00, 001x, 1x0x\}$

$g(1) = \{0000, 0100, 1000, 1100, 0010, 0011, 1000, 1001, 1100, 1101\}$

$g(1) = \{0, 2, 3, 9, 8, 9, 12, 13\}$ $g(0) = \{1, 5, 6, 7, 10, 11, 14, 15\}$

Датум

f-је нису једноке ни на једном вектору!

2. a) MIN KNF ???

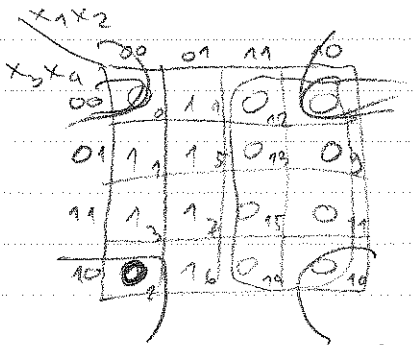
$f(x_1, x_2, x_3, x_4)$

$$f = \overline{\left(x_1 + \overline{x_2}\,\overline{x_3}\,\overline{\left(x_1 + \overline{(\overline{x_3 + x_4})}\right)\cdot(x_2 + x_3)}\right)}\cdot(x_1 + x_2 + x_4)$$

$$f = \overline{\left(x_1 + \overline{x_2}\,\overline{x_3}\cdot\left(\overline{x_1}\cdot(\cancel{x_3} + x_4)\right)\cdot(x_2 + x_3)\right)}(x_1 + x_2 + x_4)$$

$$f = \overline{\left(x_1 + \overline{x_1}\,\overline{x_2}\,\overline{x_3}\,x_4\cdot(x_2 + x_3)\right)}(x_1 + x_2 + x_4)$$

$$f = (\overline{x_1})(x_1 + x_2 + x_4) = \boxed{\overline{x_1}x_2 + \overline{x_1}x_4}$$



x_3x_4 \ x_1x_2	00	01	11	10
00	0_0	1_4	0_{12}	0_8
01	1_1	1_5	0_{13}	0_9
11	1_3	1_7	0_{15}	0_{11}
10	0_2	1_6	0_{14}	0_{10}

$f(1) = \{01xx,\ 0xx1\}$

$f(1) = \{0100, \underline{0101}, 0110, \underline{0111}, 0001, 0011, \cancel{0101}, \cancel{0111}\}$

$f(1) = \{1, 3, 4, 5, 6, 7\}$

$$\boxed{f(x_1, x_2, x_3, x_4) = \overline{x_1}\cdot(x_2 + x_4)}$$

НИС

Датум

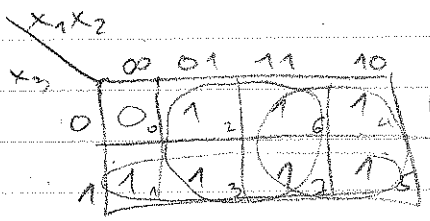
MIN DNF ???

2. a) $f(x_1, x_2, x_3) = x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 + x_1 x_2$

$f(1) = \{ xx1, 010, 10x, 11x \}$

$f(1) = \{ 001, 011, 101, 111, 010, 100, 101, 110, 111 \}$

$f(1) = \{ 1, 2, 3, 9, 5, 6, 7 \}$



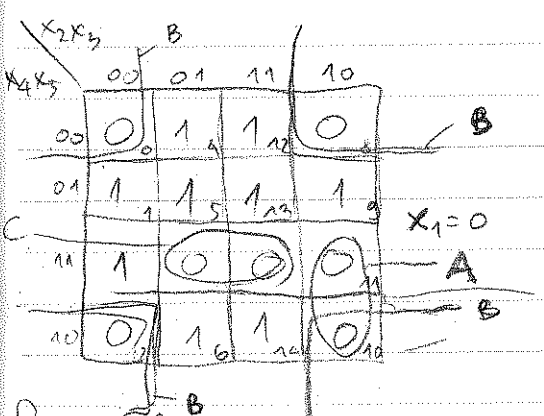
$f(x_1, x_2, x_3) = x_3 + x_2 + x_1$

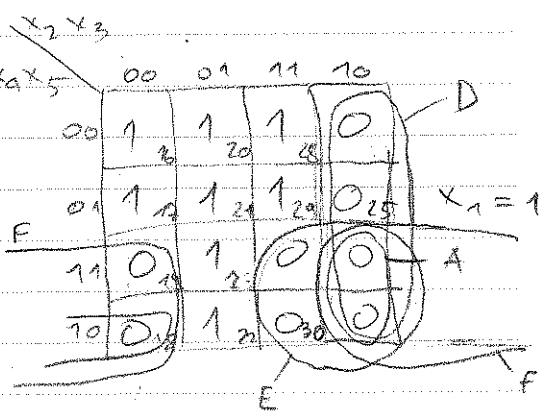
$= x_1 + x_2 + x_3$

b) MIN KNF ???

$f(x_1, x_2, x_3, x_4, x_5)$

$f(1) = \{ 1, 3, 9, 5, 6, 9, 12, 13, 14, 16, 17, 20, 21, 22, 23, 25, 29 \}$





$f(0) = \{ 0, 2, 7, 8, 10, 11, 15, 18, 19, 24, 25, 26, 27, 30, 31 \}$

$f(x_1, x_2, x_3, x_4, x_5) = A \cdot B \cdot C \cdot D \cdot E \cdot F$

$A = \bar{x}_2 + x_3 + \bar{x}_4$ $\quad C = x_1 + \bar{x}_3 + \bar{x}_4 + \bar{x}_5$ $\quad E = \bar{x}_1 + \bar{x}_2 + \bar{x}_4$

$B = x_1 + x_3 + x_5$ $\quad D = \bar{x}_1 + \bar{x}_2 + x_3$ $\quad F = \bar{x}_1 + x_3 + \bar{x}_4$

Датум

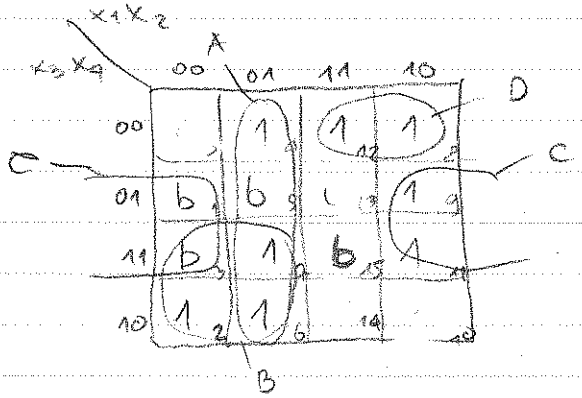
MIN DNF

2. r) $f(x_1, x_2, x_3, x_4)$

$f(0) = \{0, 10, 13, 14\}$

$f(b) = \{1, 3, 5, 15\}$

$f(1) = \{2, 4, 6, 7, 8, 9, 11, 12\}$



$A = \bar{x}_1 x_2$

$B = \bar{x}_1 x_3$

$C = \bar{x}_2 x_4$

$D = x_1 \bar{x}_3 \bar{x}_4$

$f(x_1, x_2, x_3, x_4) = A + B + C + D$

$f(x_1, x_2, x_3, x_4) = \bar{x}_1 x_2 + \bar{x}_1 x_3 + \bar{x}_2 x_4 + x_1 \bar{x}_3 \bar{x}_4$

I КОЛОКВИЈУМ 28.03.2010.

НИС ПРВИ Датум

1. а)

$f(x_1,x_2,x_3,x_4) = (x_1+x_2)\cdot(x_1+\overline{x_2}+\overline{x_3})\cdot(\overline{x_1}+x_2+\overline{x_4})(\overline{x_1}+x_3)$

$g(x_1,x_2,x_3,x_4) = \overline{x_1}\cdot x_3\cdot x_4 + x_2x_4 + \overline{x_1}x_2x_3 + x_1x_2$

$f(0) = \{00xx, 011x, 10x1, 1x0x\}$

$f(0) = \{0000, 0001, 0010, 0011, 0110, 0111, 1001, 1011, 1000, 1001, 1100, 1101\}$

$f(0) = \{0, 1, 2, (3), (6), (7), 8, 9, 11, (12), (13)\}$

$f(1) = \{(4)5, (10), 14, 15\}$

$g(1) = \{0x11, x1x1, 011x, 11xx\}$

$g(1) = \{0011, 0111, 0101, 0111, 1101, 1111, 0110, 0111, 1100, 1101, 1110, 1111\}$

$g(1) = \{(3), 5, (6), (7), (12), (13), 14, 15\}$.

$g(0) = \{0, 1, 2, (9), 8, 9, (10), 11\}$

РАЗЛИКУЈУ : $\{3, 9, 6, 7, 10, 12, 13\}$

$$\overline{(\bar{x}_1\bar{x}_2 + x_1x_3 + \bar{x}_1\bar{x}_3(x_1+\bar{x}_2)} =$$

$$\overline{(\bar{x}_1\bar{x}_2 + x_1x_3 + \bar{x}_1\bar{x}_2\bar{x}_3)} =$$

$$((x_1+x_2)(\bar{x}_1+\bar{x}_3)\cdot(x_1+x_2+x_3)$$

1. б)

SKNF ???

$$f(x_1,x_2,x_3,x_4) = (\bar{x}_1+x_2+\bar{x}_3)\overline{(\bar{x}_1\bar{x}_2 + x_1x_3 + \overline{(x_1+x_3)}(x_1+\bar{x}_2))}$$

$$f = (\bar{x}_1+x_2+\bar{x}_3)((x_1+x_2)\cdot(\bar{x}_1+\bar{x}_3)\cdot(x_1+x_3+\bar{x}_1x_2))$$

$$f = (\bar{x}_1+x_2+\bar{x}_3)((x_1\bar{x}_3+\bar{x}_1x_2+x_2\bar{x}_3)(x_1+\bar{x}_1x_2+x_3))$$

$$f = (\bar{x}_1+x_2+\bar{x}_3)(x_1\bar{x}_3+\bar{x}_1x_2+\bar{x}_1x_2x_3+x_1x_2\bar{x}_3+\bar{x}_1x_2\bar{x}_3)$$

$$f = \bar{x}_1x_2 + \cancel{\bar{x}_1x_2x_3} + \cancel{\bar{x}_1x_2\bar{x}_3} + \cancel{x_1x_2\bar{x}_3} + \cancel{\bar{x}_1x_2} + \cancel{\bar{x}_1x_2x_3} +$$
$$\cancel{x_1x_2\bar{x}_3} + \cancel{\bar{x}_1x_2\bar{x}_3} + x_1\bar{x}_3 + \cancel{\bar{x}_1x_2\bar{x}_3} + \cancel{x_1x_2\bar{x}_3} + \cancel{\bar{x}_1x_2\bar{x}_3}$$

$$f = \bar{x}_1x_2 + x_1\bar{x}_3$$

$f(1) = \{01xx, 1x0x,$

$f(1) = \{0100, 0101, 0110, 0111, 1000, 1001,$
$1100, 1101,\}$

$f(1) = \{4, 5, 6, 7, 8, 9, 12, 13\}$

$f(0) = \{0, 1, 2, 3, 10, 11, 14, 15\}$

$f(0) = \{0000, 0001, 0010, 0011, 1010, 1011,$
$1110, 1111\}$

SKNF:

$$f(x_1,x_2,x_3,x_4) = (x_1+x_2+x_3+x_4)(x_1+x_2+x_3+\bar{x}_4)$$
$$(x_1+x_2+\bar{x}_3+x_4)(x_1+x_2+\bar{x}_3+x_4)(\bar{x}_1+x_2+\bar{x}_3+x_4)$$
$$(\bar{x}_1+x_2+\bar{x}_3+\bar{x}_4)(\bar{x}_1+\bar{x}_2+\bar{x}_3+x_4)(\bar{x}_1+\bar{x}_2+\bar{x}_3+\bar{x}_4)$$

MIN KNF ???

2. a)

$$f(x_1, x_2, x_3) = x_1\bar{x}_3 + \bar{x}_2\bar{x}_3 + x_1x_2x_3$$

$$f(1) = \{1\times0,\ \times00,\ 111\}$$

$$f(1) = \{100,\ 110,\ 000,\ 100,\ 111\}$$

$$f(1) = \{0, 4, 6, 7\}$$

x_3 \ x_1x_2	00	01	11	10
0	1 (0)	0 (2)	1 (6)	1 (4)
1	0 (1)	0 (3)	1 (7)	0 (5)

MIN KNF:

$$f(x_1, x_2, x_3) = (x_1 + \bar{x}_2)(x_2 + \bar{x}_3)$$

2. d)

MIN DNF ???

$$f(x_1, x_2, x_3, x_4) = x_1 \cdot (\bar{x}_2 + \bar{x}_3 + \bar{x}_2\bar{x}_4x_1 + \bar{x}_3x_4) + x_1\bar{x}_3 \cdot x_4 + \bar{x}_1\bar{x}_3x_4 + \bar{x}_1x_2x_4$$

$$f = x_1 \cdot x_2 \cdot \bar{x}_3 \cdot (x_2\bar{x}_4 + \bar{x}_1) \cdot x_3x_4 + x_1\bar{x}_3x_4 + \bar{x}_1\bar{x}_3x_4 + \bar{x}_1x_2x_4$$

$$f = x_1x_2\bar{x}_3x_4(x_2\bar{x}_4 + \bar{x}_1) + \underbrace{x_1\bar{x}_3x_4 + \bar{x}_1\bar{x}_3x_4}_{\bar{x}_3x_4} + \bar{x}_1x_2x_4$$

$$f(1) = \{\ ,\ 1\times01,\ 0\times01,\ 01\times1\}$$

$$f(1) = \{\ ,\ 1001,\ 1101,\ 0001,\ 0101,\ 0101,\ 0111\}$$

$$f(1) = \{1, 5, 7, 9, 13,\ \}$$

x_3x_4 \ x_1x_2	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

$$f(x_1, x_2, x_3, x_4) = \bar{x}_3x_4 + \bar{x}_1x_2x_4$$

НИС

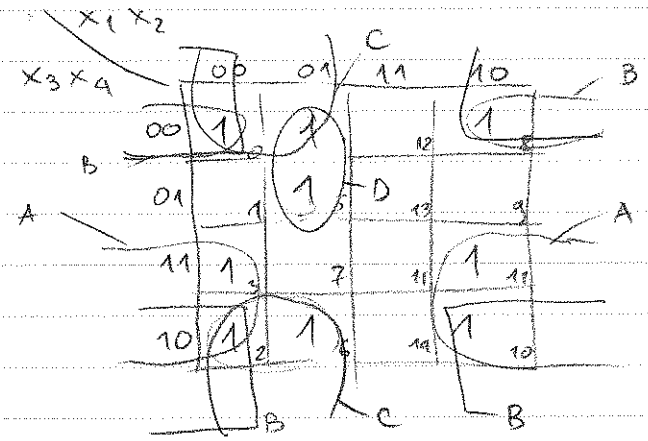
Датум

MIN DNF

2. b) $f(x_1, x_2, x_3, x_4)$

$f(0) = \{1, 7, 9, 12, 13, 14, 15\}$

$f(1) = \{0, 2, 3, 4, 5, 6, 8, 10, 11\}$



$A = \overline{x_2} x_3$

$B = \overline{x_2}\,\overline{x_4}$

$C = \overline{x_1}\,\overline{x_4}$

$D = \overline{x_1} x_2 \overline{x_3}$

$f(x_1, x_2, x_3, x_4) = A + B + C + D$

$f(x_1, x_2, x_3, x_4) = \overline{x_2} x_3 + \overline{x_2}\,\overline{x_4} + \overline{x_1}\,\overline{x_4} + \overline{x_1} x_2 \overline{x_3}$

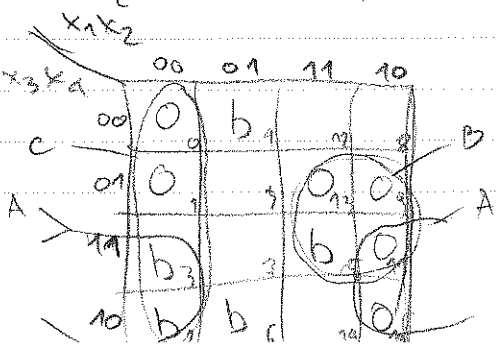
2. i) MIN KNF ???

$f(x_1, x_2, x_3, x_4)$

$f(1) = \{5, 7, 8, 12, 14\}$

$f(b) = \{2, 3, 4, 6, 15\}$

$f(0) = \{0, 1, 9, 10, 11, 13\}$



$A = x_2 + \overline{x_3}$

$B = \overline{x_1} + \overline{x_4}$

$C = x_1 + x_2$

НИС

Датум

MIN KNF

$$f(x_1, x_2, x_3, x_4) = A \cdot B \cdot C$$

$$f(x_1, x_2, x_3, x_4) = (x_2 + \overline{x_3})(\overline{x_1} + \overline{x_4})(x_1 + x_2)$$

I КОЛОКВИУМ 15.05.2010,

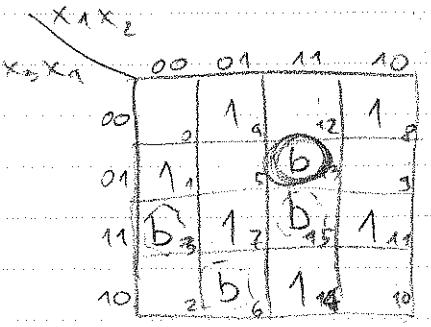
НАДОКНАДА Datum

1. a) $f(x_1, x_2, x_3, x_4)$

$f(0) = \{0, 2, 5, 9, 10, 12\}$

$f(b) = \{3, 6, 13, 15\}$

SDNF = MIN DNF $f(1) = \{1, 4, 7, 8, 11, 14\}$



1) SDNF = MIN DNF → $f(0) = \{3, 6, 13, 15\}$

2) SDNF = MIN DNF → $f(0) = \{3, 6, 15\} \wedge f(1) = \{13\}$

1. d) =

$f(x_1, x_2, x_3, x_4) = (\overline{x_1} + \overline{x_2} + \overline{x_4})(\overline{x_1} + x_3)(x_3 + x_4)$

$g(x_1, x_2, x_3, x_4) = x_2\overline{x_4} + \overline{x_2}x_3\overline{x_4} + \overline{x_1}x_3$

$f(0) = \{11x1, 1x0x, xx00\}$

$f(0) = \{1101, 1111, 1000, 1001, 1100, 1101, 0000, 0100, 1000, 1100\}$

$f(0) = \{0, 4, 8, 9, 12, 13, 15\}$

$f(1) = \{1, 2, 3, 5, 6, 7, 10, 11, 14\}$

$g(1) = \{x1x0, x010, 0x1x\}$

$g(1) = \{0100, 0110, 1100, 1110, 0010, 1010, 0010, 0011, 0110, 0111\}$

$g(1) = \{2, 3, 4, 6, 7, 10, 12, 14\}$

$g(0) = \{0, 1, 5, 8, 9, 11, 13, 15\}$

JEDNAKE: $\{0, 2, 3, 6, 7, 8, 9, 10, 13, 14, 15\}$

2. a) MIN KNF ???

$f = (x_1x_4 + \overline{x_2} \cdot (\overline{x_1 + x_3 + (x_1 + x_3 + x_4) \cdot (x_1 + x_2)})(\overline{x_2 + x_3})$

$f = (\overline{x_1x_4 + \overline{x_2}(x_1 + x_2)(\overline{x_1} \cdot \overline{x_3} \cdot (x_1 + x_3 + x_4))} \cdot x_2 \cdot \overline{x_3}$

$f = (\overline{x_1x_4 + (x_1\overline{x_2})(\overline{x_1}\,\overline{x_3}\,x_4)})x_2\overline{x_3}$

$f = (\overline{x_1x_4})x_2\overline{x_3} = (\overline{x_1} + \overline{x_4})x_2\overline{x_3}$

$f = \overline{x_1}x_2\overline{x_3} + x_2\overline{x_3}\,\overline{x_4}$

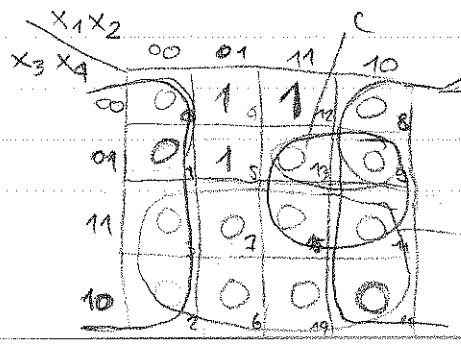
$f(1) = \{010x, x100\} = \{0100, 0101, 0100, 1100\}$

$f(1) = \{4, 5, 12\}$

MIN KNF

$f(x_1, x_2, x_3, x_4) = A \cdot B \cdot C$

$f = x_2 \cdot \overline{x_3}(\overline{x_1} + \overline{x_4})$



Датум

MIN DNF

2.б) $f(x_1,x_2,x_3) = x_3 + \bar{x}_1 x_2 \bar{x}_3 + \underbrace{x_1\bar{x}_2 + x_1 x_2}_{x_1(\underbrace{x_2+\bar{x}_2}_{1}) = x_1}$

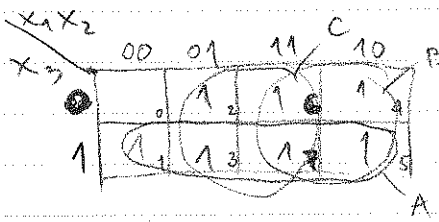
$f(1) = \{xx1,\ 010,\ 1xx\}$

$f(1) = \{001, 011, \underline{101}, \cancel{111}, 010, 100, \cancel{101}, 110, \underline{111}\}$

$f(1) = \{1, 2, 3, 4, 5, 6, 7\}$

x_3 \ x_1x_2	00	01	11	10
0	0	1 2	1 6	1 4
1	1 1	1 3	1 7	1 5

A, B, C



$A = x_3$

$B = x_1$

$C = x_2$

$f(x_1,x_2,x_3) = A + B + C$

$f(x_1,x_2,x_3) = x_1 + x_2 + x_3$

---

2. в) MIN KNF???

$f(x_1,x_2,x_3,x_4)$

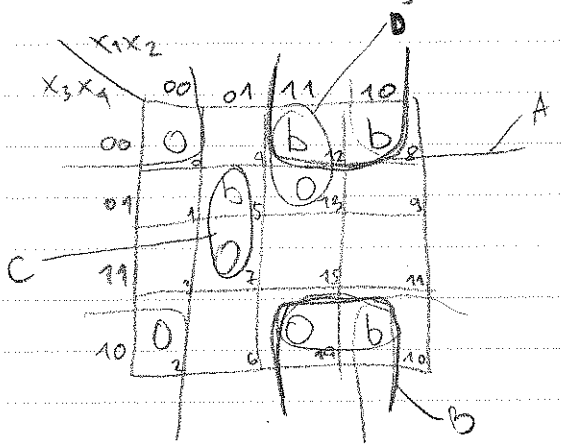
$f(1) = \{1, 3, 4, 6, 9, 11, 15\}$

$f(b) = \{5, 8, 10, 12\}$

$f(0) = \{0, 2, 7, 13, 14\}$

x_3x_4 \ x_1x_2	00	01	11	10
00	0 0		b 4	b 12 8
01	1	b 5	0 13	9
11	3	0 7	15	11
10	0 2		0 14	b 10

A, B, C, D



$A = x_2 + x_4$

$B = \bar{x}_1 + x_4$

$C = x_1 + \bar{x}_2 + \bar{x}_4$

$D = \bar{x}_1 + \bar{x}_2 + x_3$

$f(x_1,x_2,x_3,x_4) = A \cdot B \cdot C \cdot D$

$f(x_1,x_2,x_3,x_4) = (x_2 + x_4)(\bar{x}_1 + x_4)(x_1 + \bar{x}_2 + \bar{x}_4)(\bar{x}_1 + \bar{x}_2 + x_3)$

Датум

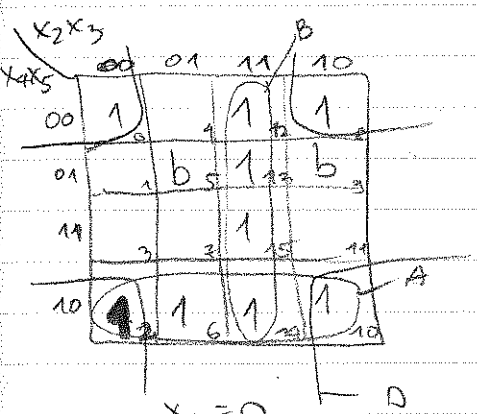
1. а) MIN DNF

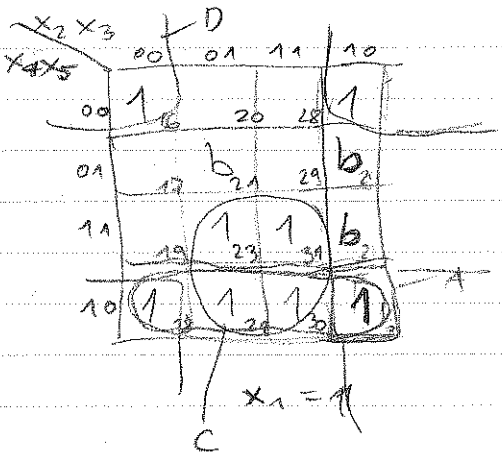
$f(x_1, x_2, x_3, x_4, x_5)$

$f(0) = \{1, 3, 4, 7, 11, 17, 19, 20, 28, 29\}$

$f(1) = \{0, 2, 6, 8, 10, 12, 13, 14, 15, 16, 18, 22, 23, 24, 26, 30, 31\}$

$f(b) = \{5, 9, 21, 25, 27\}$





$f(x_1, x_2, x_3, x_4, x_5) = A + B + C + D$

$A = x_4 \cdot \overline{x_5}$

$B = \overline{x_1}\, x_2 x_3$

$C = x_1 x_3 x_4$

$D = \overline{x_3}\, \overline{x_5}$

$$f(x_1, x_2, x_3, x_4, x_5) = x_4 \overline{x_5} + \overline{x_1} x_2 x_3 + x_1 x_3 x_4 + \overline{x_3}\, \overline{x_5}$$

I КОЛОКВИЈУМ 20.3.2011.

НИС ПРВИ Датум

1. а) ≠

$f(x_1, x_2, x_3, x_4) = (x_1 + x_2 + \overline{x_4}) \cdot (\overline{x_1} + \overline{x_3} + x_4) \cdot (\overline{x_2} + \overline{x_3} + x_4) \cdot (\overline{x_1} + \overline{x_2} + x_3)$

$g(x_1, x_2, x_3, x_4) = \overline{x_1} \cdot \overline{x_2}\overline{x_4} + \overline{x_1}x_2x_4 + x_1\overline{x_2}\overline{x_3} + x_1 \cdot x_2$

$f(0) = \{00x1, 1x10, x110, 110x\}$

$f(0) = \{0001, 0011, 1010, 1110, 0110, 1100, 1101\}$

$f(0) = \{1, 3, 6, 10, 12, 13, 14\}$

$f(1) = \{0, 2, 4, 5, 7, 8, 9, 11, 15\}$

$g(1) = \{00x0, 01x1, 100x, 11xx\}$

$g(1) = \{0000, 0010, 0101, 0111, 1000, 1001, 1100, 1101, 1110, 1111\}$

$g(1) = \{0, 2, 5, 7, 8, 9, 12, 13, 14, 15\}$

$g(0) = \{1, 3, 4, 6, 10, 11\}$

RAZLIKUJU SE: $\{4, 11, 12, 13, 14\}$

Datum

SKNF

1. d) $f(x_1, x_2, x_3, x_4)$

$$f = \overline{(\overline{x_1} + x_2 + \overline{x_1}x_2\overline{x_4})} + \overline{(\overline{x_2}\overline{x_4} + x_2x_3)}$$

$$f = (\overline{x_1}+x_2)\cdot(x_1+x_2+x_4) + (x_2+x_4)\cdot(\overline{x_2}+\overline{x_3})$$

$$f = \overline{x_1}x_2 + \overline{x_1}x_4 + x_1x_2 + x_2 + \overline{x_2}x_4 + x_2\overline{x_3} + \overline{x_2}x_4 + \overline{x_3}x_4$$

$$f = \overline{x_1}x_4 + \overline{x_2}x_4 + \overline{x_3}x_4 + x_2$$

$f(1) = \{0\times\times1,\ \times0\times1,\ \times\times01,\ \times1\times\times\}$

$f(1) = \{0001, 0011, 0101, 0111, 0001, 0011, 1001, 1011, 0001, 0101, 1001, 1101, 0100, 0101, 0110, 0111, 1100, 1101, 1110, 1111\}$

$f(1) = \{1, 3, 5, 7, 9, 6, 9, 11, 12, 13, 14, 15\}$

$f(1) = \{1, 3, 9, 5, 6, 7, 9, 11, 12, 13, 14, 15\}$

$f(0) = \{0, 2, 8, 10\} = \{0000, 0010, 1000, 1010\}$

SKNF:

$$f(x_1, x_2, x_3, x_4) = (x_1+x_2+x_3+x_4)(x_1+x_2+\overline{x_3}+x_4)(\overline{x_1}+x_2+x_3+x_4)(\overline{x_1}+x_2+\overline{x_3}+x_4)$$

Датум

MIN KNF ???

2. a) $f(x_1, x_2, x_3, x_4)$

$$f = \left(x_2\overline{x_3} + \overline{\left(\overline{\overline{x_1}\cdot\left(\overline{x_2+\overline{x_3}+x_4}\right)} + \overline{\left(x_1+\overline{x_4}\right)} + \overline{x_3\cdot\left(\overline{x_2+x_3+x_4}\right)}\right)}\right)$$

$$f = \left(x_2\overline{x_3} + \overline{\left(x_1 + \left(x_2+\overline{x_3}+x_4\right) + \overline{x_1}\cdot x_4 + \cancel{\overline{x_3}} + x_2\overline{x_3}\,\overline{x_4}\right)}\right)$$

$$f = \left(x_2\overline{x_3} + \overline{\left(x_1 + x_2 + \overline{x_3} + x_4 + \cancel{\overline{x_1}\,x_4} + \cancel{x_2\overline{x_3}\,\overline{x_4}}\right)}\right)$$

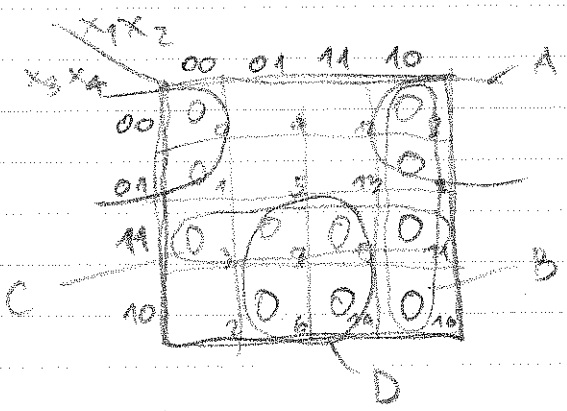
$$f = \left(x_2\overline{x_3} + \overline{\left(x_1 + x_2 + \overline{x_3} + x_4\right)}\right)$$

$$f = x_2\overline{x_3} + \overline{x_1}\,\overline{x_2}\,x_3\,\overline{x_4}$$

$f(1) = \{x10x, 0010\}$

$f(1) = \{0100, 0101, 1100, 1101, 0010\}$

$f(1) = \{2, 4, 5, 12, 13\}$



$A = x_2 + x_3$

$B = \overline{x_1} + x_2$

$C = \overline{x_3} + \overline{x_4}$

$D = \overline{x_2} + \overline{x_3}$

$f(x_1, x_2, x_3, x_4) = A \cdot B \cdot C \cdot D$

$f(x_1, x_2, x_3, x_4) = (x_2 + x_3)(\overline{x_1} + x_2)(\overline{x_3} + \overline{x_4})(\overline{x_2} + \overline{x_3})$

НИС

Датум .....

MIN DNF ???

2. a)

$f(x_1, x_2, x_3)$

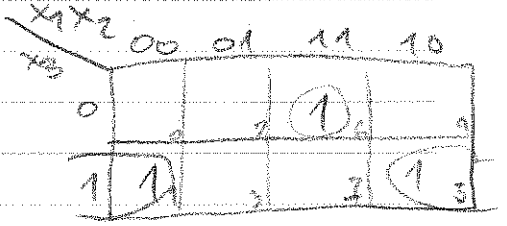
$f = (x_1 + x_3)(\overline{x_2} + \overline{x_3})(x_1 + x_2 + x_3)(\overline{x_1} + x_2 + x_3)$

$f(0) = \{0x0, x11, \cancel{000}, 100\}$

$f(0) = \{000, 010, 011, 111, 100\}$

$f(0) = \{0, 2, 3, 7, 4\}$

$f(1) = \{1, 5, 6\}$



MIN DNF:

$f(x_1, x_2, x_3) = \overline{x_2}x_3 + x_1x_2\overline{x_3}$

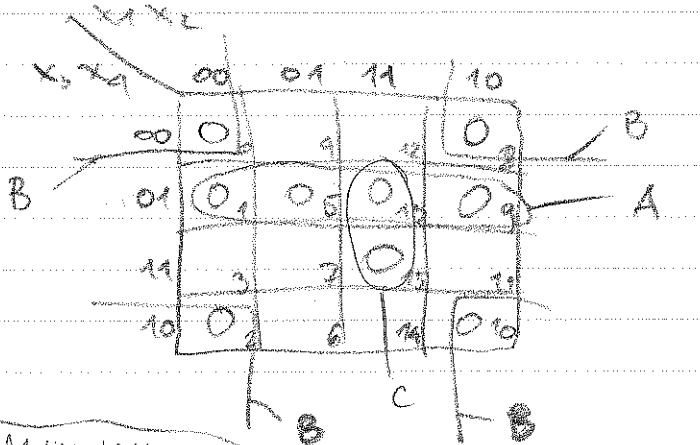
2. b)

MIN KNF ???

$f(x_1, x_2, x_3, x_4)$

$f(1) = \{3, 9, 6, 7, 11, 12, 14\}$

$f(0) = \{0, 1, 2, 5, 8, 13, 10, 13, 15\}$



$A = x_3 + \overline{x_4}$

$B = x_2 + x_4$

$C = \overline{x_1} + \overline{x_2} + \overline{x_4}$

MIN KNF

$f(x_1, x_2, x_3, x_4) = A \cdot B \cdot C = (x_2 + x_4)(x_3 + \overline{x_4})(\overline{x_1} + \overline{x_2} + \overline{x_4})$

Datum

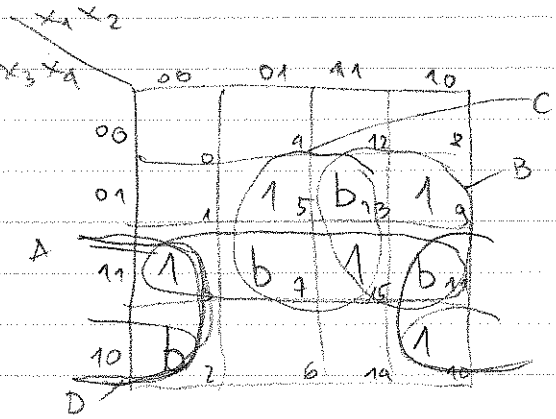
MIN DNF ???

2. i) $f(x_1, x_2, x_3, x_4)$

$f(0) = \{0, 1, 4, 6, 8, 12, 14\}$

$f(b) = \{2, 7, 11, 13\}$

$f(1) = \{3, 5, 9, 10, 15\}$



~~A~~ = 0

$B = x_1 x_4$

$C = x_2 x_4$

$D = \bar{x}_2 x_3$

$f(x_1, x_2, x_3, x_4) = \cancel{A} + B + C + D$

$f(x_1, x_2, x_3, x_4) = x_1 x_4 + x_2 x_4 + \bar{x}_2 x_3$

I КОЛОКВИЈУМ 08.05.2011.

НАДОКНАДА Датум

1. a)

$f(x_1, x_2, x_3, x_4) = (\bar{x}_1 + \bar{x}_2 + x_3)\cdot(x_2 + x_3 + x_4)(x_1 + x_2 + \bar{x}_3)(x_2 + \bar{x}_3 + \bar{x}_4)$

$f(0) = \{110x, x000, 001x, x011\}$

$f(0) = \{1100, 1101, 0000, 1000, 0010, 0011, 0011, 1011\}$

$f(0) = \{0, 2, 3, 8, 11, 12, 13\}$

$f(1) = \{1, 4, 5, 6, 7, 9, 10, 14, 15\}$

$g(x_1, x_2, x_3, x_4) = x_3\bar{x}_4 + x_2x_3 + \bar{x}_1x_2\bar{x}_4 + x_1\bar{x}_2\bar{x}_3x_4$

$g(1) = \{xx10, x11x, 01x0, 1001\}$

$g(1) = \{0010, 0110, 1010, 1110, 0110, 0111, 1110, 1111, 0100, 0110, 1001\}$

$g(1) = \{2, 4, 6, 7, 9, 10, 14, 15\}$

$g(0) = \{0, 1, 3, 5, 8, 11, 12, 13\}$

JEDNAKE: $\{0, 3, 4, 6, 7, 8,9, 10, 11, 12, 13, 14, 15\}$

НИС

Датум

SDNF

1. б) $f(x_1, x_2, x_3, x_4)$

$$f = \overline{(\overline{x_1} + x_2 + \overline{x_1 \cdot \overline{x_2} \cdot \overline{x_4}})} + \overline{(\overline{x_2 \cdot \overline{x_4}} + \overline{x_2 \cdot x_3})}$$

$$f = \overline{(\overline{x_1} + x_2 + \overline{(x_1 + x_2)\overline{x_4}})} + \overline{((x_2 + x_4) \cdot (x_2 x_3))}$$

$$f = (x_1 \cdot \overline{x_2} \cdot \overline{x_4}(x_1 + x_2)) + x_2 x_3 + x_2 x_3 x_4$$

$$f = x_1 \overline{x_2}\,\overline{x_4} + x_2 x_3$$

$$f(1) = \{10x0, x11x\} = \{1000, 1010, 0110, 0111, 1110, 1111\}$$

$$f(1) = \{6, 7, 8, 10, 14, 15\}$$

$$f(0) = \{0, 1, 2, 3, 4, 5, 9, 11, 12, 13\}$$

SDNF

$$f(x_1, x_2, x_3, x_4) = x_1 \overline{x_2} \overline{x_3} \overline{x_4} + x_1 \overline{x_2} x_3 \overline{x_4} + \overline{x_1} x_2 x_3 \overline{x_4} + \overline{x_1} x_2 x_3 x_4 + x_1 x_2 x_3 \overline{x_4} + x_1 x_2 x_3 x_4$$

НИС

MIN DNF ???

2. a) $f(x_1, x_2, x_3, x_4)$

$$f = \overline{(x_1 \bar{x}_2 x_3)} \cdot \overline{(\bar{x}_3 + x_4(\bar{x}_2 + \bar{x}_1))} \cdot \overline{(x_2 + x_3)} \cdot$$

$$\overline{((\overline{(x_1 + \bar{x}_4)}(\bar{x}_3 + x_2)) + x_2(\overline{x_3 + \bar{x}_1 + x_4}))}$$

$$f = (\bar{x}_1 + x_2 + \bar{x}_3)\overline{(x_3 \cdot (\overline{\bar{x}_4 + x_2 + \bar{x}_1}))} \cdot \bar{x}_2 \bar{x}_3 \cdot$$

$$\overline{((\overline{\bar{x}_1 x_4 + x_3 \cdot \bar{x}_2}) + x_2 \bar{x}_3 \cdot x_1 \cdot \bar{x}_4)}$$

$$f = (\bar{x}_1 + x_2 + \bar{x}_3)(\overline{x_3 + x_4 \bar{x}_2 x_1 + x_2 + x_3})(\overline{x_1 + \bar{x}_4})(\overline{x_2 + \bar{x}_3})$$

$$f = (\bar{x}_1 + x_2 + \bar{x}_3)((x_1 x_2 + x_1 \bar{x}_3 + x_2 \bar{x}_4 + \bar{x}_3 \bar{x}_4)(\overline{\bar{x}_1 + \bar{x}_2 + x_3 + x_4}))$$

$$f = (\bar{x}_1 + x_2 + \bar{x}_3)(x_1 x_2 x_3 + x_1 x_2 x_4 + x_1 \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_3 x_4 + \bar{x}_1 x_2 \bar{x}_4 + x_2 x_3 \bar{x}_4 + \bar{x}_1 \bar{x}_3 \bar{x}_4 + \bar{x}_2 \bar{x}_3 \bar{x}_4)$$

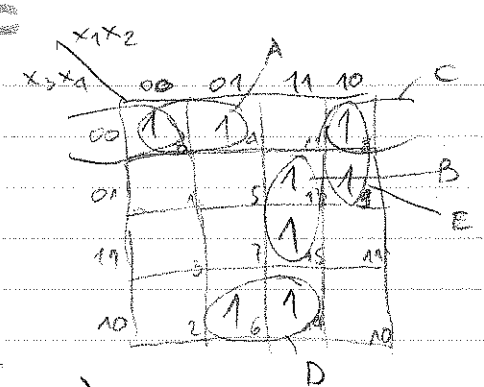
$$f = \bar{x}_1 x_2 \bar{x}_4 + \bar{x}_1 x_2 x_3 x_4 + \bar{x}_1 \bar{x}_3 \bar{x}_4 + \bar{x}_1 x_2 \bar{x}_3 \bar{x}_4 + x_1 x_2 x_3 + x_1 x_2 x_4 + x_1 x_2 \bar{x}_3 x_4 + \bar{x}_1 x_2 \bar{x}_4 + x_2 x_3 \bar{x}_4 + x_1 \bar{x}_2 x_3 \bar{x}_4 + x_1 \bar{x}_2 \bar{x}_3 x_4 + x_1 \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_3 x_4 + \bar{x}_2 \bar{x}_3 \bar{x}_4$$

$$f = \bar{x}_1 x_2 \bar{x}_4 + \bar{x}_1 \bar{x}_3 \bar{x}_4 + x_1 x_2 x_3 + x_1 x_2 x_4 + x_2 x_3 \bar{x}_4 + x_1 \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_3 x_4 + \bar{x}_2 \bar{x}_3 \bar{x}_4$$

$f(1) = \{01x0, 0x00, 111x, 11x1, x110, 100x, 1x01, x000\} = \{0100, 0110, 0000, 0100, 110, 1111, 1101, 1111, 0110, 1110, 1000, 1001, 1001, 1101, 1000, 0000\} = \{0, 4, 6, 8, 9, 13, 14, 15\}$

НИС

Датум



$A = \bar{x}_1 \bar{x}_3 \bar{x}_4$

$B = x_1 x_2 x_4$

$C = \bar{x}_2 \bar{x}_3 \bar{x}_4$

$D = x_2 x_3 \bar{x}_4$

$E = x_1 \bar{x}_2 \bar{x}_3$

$f(x_1, x_2, x_3, x_4) = A + B + C + D + E$

$f(x_1, x_2, x_3, x_4) = \bar{x}_1 \bar{x}_3 \bar{x}_4 + x_1 x_2 x_4 + \bar{x}_2 \bar{x}_3 \bar{x}_4 + x_2 x_3 \bar{x}_4 + x_1 \bar{x}_2 \bar{x}_3$

MIN KNF ???

б)

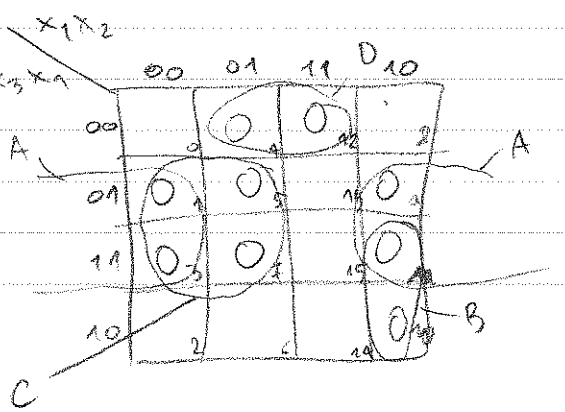
$f(x_1, x_2, x_3, x_4) = \bar{x}_1 x_3 \bar{x}_4 + \bar{x}_2 \bar{x}_3 \bar{x}_4 + x_1 x_2 \bar{x}_3 x_4 + \bar{x}_1 \bar{x}_2 \bar{x}_4 + x_2 x_3 \bar{x}_4 + x_1 x_2 x_3$

$f(1) = \{0x10, x000, 1101, 00x0, x110, 111x\}$

$f(1) = \{0010, 0110, 0000, 1000, 1101, 0000, 0010, 0110, 1110, 1110, 1111\}$

$f(1) = \{0, 2, 6, 8, 13, 19, 15\}$

$f(0) = \{1, 3, 4, 5, 7, 9, 10, 11, 12\}$



$A = x_2 + \bar{x}_4$

$B = \bar{x}_1 + x_2 + \bar{x}_3$

$C = x_1 + \bar{x}_4$

$D = \bar{x}_2 + x_3 + x_4$

НИС

Datum .....

MIN KNF ???

$f(x_1, x_2, x_3, x_4) = A \cdot B \cdot C \cdot D$

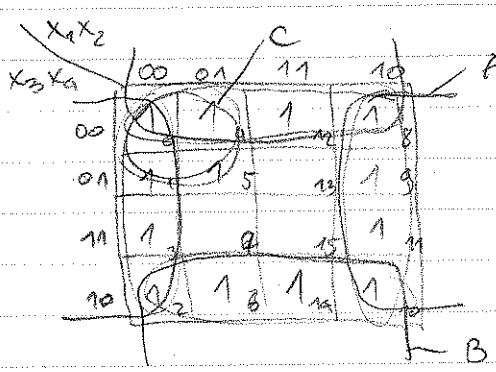
$f(x_1, x_2, x_3, x_4) = (x_2 + \overline{x_4})(\overline{x_1} + x_2 + \overline{x_3})(x_1 + \overline{x_4})(\overline{x_2} + x_3 + x_4)$

2.b) MIN DNF ???

$f(x_1, x_2, x_3, x_4)$

$f(0) = \{7, 13, 15\}$

$f(1) = \{0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14\}$



$A = \overline{x_2}$

$B = \overline{x_4}$

$C = \overline{x_1}\,\overline{x_3}$

$f(x_1, x_2, x_3, x_4) = A + B + C$

$f(x_1, x_2, x_3, x_4) = \overline{x_2} + \overline{x_4} + \overline{x_1}\,\overline{x_3}$ MIN DNF

НИС

Датум

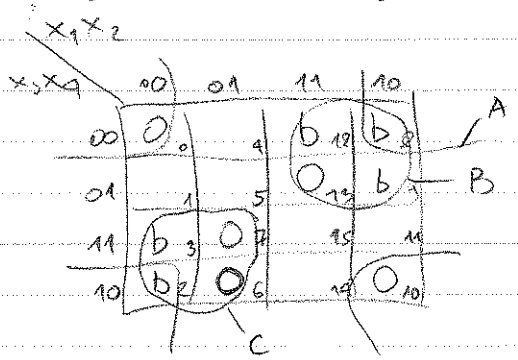
MIN KNF ???

1) $f(x_1, x_2, x_3, x_4)$

$f(1) = \{1, 9, 5, 11, 14, 15\}$

$f(b) = \{2, 3, 8, 9, 12\}$

$f(0) = \{0, 6, 7, 10, 13\}$



$A = x_2 + x_4$

$B = \bar{x}_1 + x_3$

$C = x_1 + \bar{x}_3$

$f(x_1, x_2, x_3, x_4) = A \cdot B \cdot C$

$f(x_1, x_2, x_3, x_4) = (x_2 + x_4)(\bar{x}_1 + x_3)(x_1 + \bar{x}_3)$ MIN KNF